Peyresq 2009 June 21-25

Cedric Deffayet, "Covariant Galileon" 0901.1314, 0906.1967

This model was proposed by Nicolis, Rattazzi, Trincherini 0811.2197 from DGP model $M_{(5)}^3 \int d^5x \sqrt{g}\, R_{(5)} + \int d^4x \sqrt{g}\, \mathcal{L}_m(g, \psi_m) + M_P^2 \int d^4x \sqrt{g}\, R$.
${}^{(4)}R = K^2 - K_{\mu\nu}K^{\mu\nu}$, $K_{\mu\nu} = -\frac{1}{M_{(5)}^3}\left(\frac{1}{2}S_{\mu\nu} - \frac{1}{6}S g_{\mu\nu}\right)$, $S_{\mu\nu} = T_{\mu\nu}^{\text{matter}} - M_P^2 G_{\mu\nu}$.
Then $\frac{3}{r_c}K - K^2 + K_{\mu\nu}K^{\mu\nu} = \frac{T}{M_P^2}$, $r_c = \frac{M_P^2}{2M_{(5)}^3} \sim H_0^{-1}$. $V(r) \sim \frac{1}{r}$ for $r \ll r_c$, $\sim \frac{1}{r^2}$ for $r \gg r_c$

If I take $K_{\mu\nu} = k g_{\mu\nu}$, $g_{\mu\nu}$ = intrinsic metric on brane, $T = 0$, $\frac{12k}{r_c} = 12k^2$ with two solutions, $k = 0$ (flat brane) and $k = \frac{1}{r_c}$ (dS).

If $K_{\mu\nu} = \frac{r_c}{M_P}\nabla_\mu\nabla_\nu \pi = \frac{r_c}{M_P}\partial_\mu\partial_\nu\pi$, $3\Box\pi - \frac{1}{\Lambda^3}\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right) = \frac{T}{M_P}$,
$\Lambda = (M_P r_c^{-2})^{1/3}$. Decoupling limit (DL) of DGP: keep Λ fixed, $\frac{T}{M_P}$ fixed, $M_P \to \infty$, $r_c \to \infty$. Equation for π comes from an action
$\int d^4x\left(\frac{3}{2}\pi\Box\pi + \frac{1}{\Lambda^3}(\Box\pi\, \partial^\mu\pi\, \partial_\mu\pi) - \frac{2\pi T}{M_P^2}\right)$. $K_{\mu\nu} = k g_{\mu\nu} \Rightarrow \pi = C x^\mu x_\mu$.

Galileon generalizes this action and 2nd-order equation.
$\int d^4x \sqrt{-g}\,(1-2\pi)R + \mathcal{L}_\pi + \mathcal{L}_m(g)$. Choose $\mathcal{L}_\pi$ so $\pi = C x^\mu x_\mu$ if $g_{\mu\nu}$ is flat.
Want a self-accelerating solution, in Jordan frame $g \to$ coupled to matter. $h_{\mu\nu} = \hat{h}_{\mu\nu} + 2\pi\eta_{\mu\nu}$ where $\hat{h}_{\mu\nu}$ is Einstein frame part (over flat spacetime). deSitter is approx.
$ds^2 = \left(1 - \frac{1}{2}H^2 x^2) + \frac{1}{2}(2\dot{H} + H^2)t^2 + \cdots\right)(-dt^2 + d\vec{x}^2) = (1 - H^2 x^\mu x_\mu)(-dt^2 + d\vec{x}^2)$.
Want equation of motion to be second order. The claim of 0811.2197 is that they have found all these models for scalars in flat spacetime:
Scalar π models which can <u>only</u> contain second derivatives.

$$\mathcal{L}_{(n+1,0)} = \underset{\text{sig. of permutation}}{\sum_{\sigma\in S_n}} \epsilon(\sigma)\sum_{i=1} \pi^{,\mu_{\sigma(1)}}{}_{\mu_1}\, \pi^{\mu_{\sigma(2)}}{}_{\mu_2} \cdots \pi^{\mu_{\sigma(i-1)}}{}_{\mu(i-1)}\, \pi^{,\mu_{\sigma(i)}} \times \pi_{,\mu_i} \times \pi^{,\mu\sigma(i+1)}{}_{\mu_{i+1}} \cdots \pi^{,\mu\sigma(n)}{}_{\mu_n}.$$

In 4D, the only nontrivial models are $\mathcal{L}_{(2,0)} = \partial_\mu\pi\,\partial^\mu\pi$,
$\mathcal{L}_{(3,0)} = \partial_\mu\pi\,\partial^\mu\pi\,\Box\pi$, $\mathcal{L}_{(4,0)} = (\Box\pi)^2\partial_\mu\pi\,\partial^\mu\pi - 2(\Box\pi)\partial_\mu\pi\,\partial^\mu\partial^\nu\pi\,\partial_\nu\pi - (\partial_\mu\partial_\nu\pi)(\partial^\mu\partial^\nu\pi)(\partial_\rho\pi\,\partial^\rho\pi) + 2\partial_\mu\pi\,\partial^\mu\partial^\nu\pi\,\partial_\nu\partial_\rho\pi\,\partial^\rho\pi$, $\mathcal{L}_{(5,0)} = \cdots$

Use $\epsilon^{\mu_1\cdots\mu_d} = -\frac{1}{\sqrt{g}}\delta_1^{[\mu_1}\cdots\delta_d^{\mu_d]}$. $\sum_{\sigma S_d}\epsilon\, g^{\mu_1\nu_{\sigma(1)}}\cdots g^{\mu_d\nu_{\sigma(d)}} = -\varepsilon^{\mu_1\cdots\mu_4}\varepsilon^{\nu_1\cdots\nu_4}$.
$A_{(2n)} = \varepsilon^{1357\cdots\mu_1\mu_2\cdots\mu_{2k}}\epsilon^{246\cdots}{}_{\mu_1\cdots}$. Then $\mathcal{L}_{(3,0)} = \epsilon^{13\mu\nu}\varepsilon^{24}{}_{\mu\nu}\pi_{,1}\pi_{,2}\pi_{,34}$,
$\mathcal{L}_{(4,0)} = \epsilon^{135\mu}\epsilon^{246}{}_\mu\pi_{,1}\pi_{,2}\pi_{,34}\pi_{,56}$, $\mathcal{L}_{(5,0)} = \epsilon^{1357}\epsilon^{2468}\pi_{,1}\pi_{,2}\pi_{,34}\pi_{,56}\pi_{,78}$.
where 1 is shorthand for μ_1, etc.

In curved spacetime, naively one gets 3rd order derivatives of metric $\mathcal{L}_{(4,0)} = \epsilon^{135\mu}\epsilon^{246}{}_{\mu}\pi_{;2}\pi_{;2}\pi_{;34}\pi_{;56} \to \pi$ eom $-(\pi_{;\mu}\pi^{;\mu})(\pi_{;\nu}R^{;\nu}) +$ $+ 2(\pi_{;\mu}\pi_{;\nu}\pi_{;\rho}R^{\mu\nu;\rho})$ and $T^{\mu\nu} = (\pi_{;\lambda}\pi_{;\rho}\pi_{;\sigma}\pi^{;\lambda\rho\sigma})g^{\mu\nu} - (\pi_{;\lambda}\pi^{;\lambda})(\pi_{;\rho}\pi_{;\sigma}{}^{\sigma\rho})g^{\mu\nu}$. You can suppress these terms by $t_{n,1} = \pi_{;\mu}\pi^{;\mu}\pi_{;\nu}\pi^{;\nu}R$ and $t_{n,2} = (\pi_{;\lambda}\pi^{;\lambda}\pi_{;\mu}R^{\mu\nu}\pi_{;\nu})$. $\int d^4x\sqrt{g}(\pi_{;\mu}G^{\mu\nu}\pi_{;\nu})(\pi_{;\lambda}\pi^{;\lambda}) = \mathcal{L}_{(4,1)}$ suppresses both higher derivatives by a single term.

You can do the same for $\mathcal{L}_{(5)}$ as well. $t_{5,1} = \pi_{;\lambda}\pi^{;\lambda}\pi_{;\mu}\pi_{;\nu}\pi_{;\rho\sigma}R^{\mu\rho}$, $t_{5,2} = \pi_{;\mu}\pi^{;\mu}\pi_{;\nu}\pi^{;\nu}\pi_{;\rho\sigma}R^{\rho\sigma}$, … $t_{5,7}$. $S_5 = \int d^4x\sqrt{g}(-3t_{5,2} - 18t_{5,3} + 3t_{5,4} + \frac{15}{2}t_{5,7})$

With Stanley Deser we generalized this to higher dimensions. $\mathcal{E}\mathcal{E}\pi_{;1}\pi_{;2}(\pi_{;34}\pi_{;56} - \frac{3}{4}R_{3546}(\pi_{;a}\pi^{;a}))\pi_{;78} = \mathcal{L}_{(5,0)} + \mathcal{L}_{(5,1)}$. $\mathcal{L}_{(4,0)} + \mathcal{L}_{(4,1)} = \mathcal{E}\mathcal{E}\pi_{;1}\pi_{;2}(\pi_{;34}\pi_{;56} - \frac{1}{5}R_{3546}\pi^{;a}\pi_{;a})$. In 4D, $\mathcal{L}_{(3,0)}$ does not give problems. $\Box\pi\,\partial_\mu\pi\,\partial^\mu\pi \to (\partial\partial\pi - \partial g\partial\pi)\cdots$

$\mathcal{L}_{(n+1,0)} = A\pi_{;1}\pi_{;2}\pi_{;34}\pi_{;56}\cdots$ $\mathcal{L}_{(n+1,p)} = A\pi_{;1}\pi_{;2}\mathcal{R}_{(p)}\mathcal{S}_{(q)}$ $\mathcal{R}_{(p)} = (\pi^{;\lambda}\pi_{;\lambda})\prod_{i=1}^{p} R_{\mu_{n_i-1}\mu_{n_i+1}\mu_{n_i}\mu_{n_i+2}}$, $\mathcal{S}_{(q)} = \prod_{i=0}^{i=q-1}\pi_{;\mu_{2n-1-2i}\mu_{2n-2i}}$, $2p+q = n-1$. $\mathcal{L} = \sum_{p=0}^{p_{max}}\mathcal{C}_{(n+1,p)}\mathcal{L}_{(n+1,p)}$, $\mathcal{C}_{(n+1,p)} = \left(-\frac{1}{8}\right)^{p}\binom{n-1}{2p}\binom{2p}{p}$. This is our solution, the solution.

It is called the Galileon because it is invariant under $\partial_\mu c^\mu + b$, like $x \to x +$. [Larry Ford: Perhaps it should be called affine.] I am not responsible for the name.

Hillary Sanctuary, "Extracting the 3- and 4-Graviton Vertices from Pulsars and Coalescing Binaries" with Umberto Cannella, Michele Maggiore, and Riccardo Sturani, University of Geneva,

Motivation: Use a recent reformalization of the PN expansion of GR in terms of Feynman diagrams to directly test the 3- and 4-graviton vertices with experiment. Result: 3 graviton vertex agrees with GR to GR is non-linear, with  and . PN is post-Newtonian power series in v/c. GW experiments need high order PN, sensitive to v^6. Heroic PN calculations have been performed for GW observable [Blanchet, Dam Effective field theory method: NRGR, a reformulation on PN correcti in terms of Feynman diagrams [Goldberger and Rothstein 2004]. Non-Relativistic General Relativity, a systematic approach for PN approximations, Feynman diagrams, for binary systems, using manifest power counting. Main ingredients: Relevant scales r_s = size of compact object, orbital radius r, $\lambda = r/v$ wavelength. $\frac{r_s}{r} \sim v^2$, $\frac{r}{\lambda} \sim v$. Progressively get rid of the different scales. $S^{f}_{EH}[g^{f}_{\mu\nu}] \to S_{eff}[g_{\mu\nu}] \to S_{NR}[h_{\mu\nu}]$. We go from...

First length scale: r_s. $S_{eff}[x^\mu, g_{\mu\nu}] = S_{EH}[g] + S_{pp}[x, g]$, $g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{Pl}}$ $= \eta_{\mu\nu} + \frac{1}{M_{Pl}}(H_{\mu\nu} + \bar{h}_{\mu\nu})$, with potential gravitons $H_{\mu\nu}$ and radiation gravitons $\bar{h}_{\mu\nu}$. But the components of H and h do not scale homogeneously in powers of v. Second length scale: r. $\exp(iS_{NR}[h, x_a]/\hbar) = \cdots$ Once we have established an EFT with a clear velocity power-counting scheme, we can use the langu of Feynman diagrams to calculate PN corrections. The appropriate Feynman diagrams are tree level since classical. Potential gravitons appe only as internal lines. Radiation gravitons appear only as external lines.

NRGR has been verified up to 2PN: $L^{GR}_{cons} = [L^{Newton}]v^0 + [L^{EIH}]v^2 + [L^{2PN}]v^4 +$ $= i\int dt \frac{G_N m_1 m_2}{r} \sim mvr = L$. PN corrections to the potential Lagrangian, of order 1 scale as Lv^{2n}.

$L = L_0 + \left(\frac{1}{c^2}\right) L_2$. $L_0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{G m_1 m_2}{r}$, $L_2 = \cdots$

PN corrections to the radiation Lagrangian, of order n, scale as $L^{1/2} v^{-n+1/2}$.

strange standard

Quadrupole radiation formula $P = \frac{G_N}{5} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$, $Q_{ij} = \sum_a m_a (r_{ai} r_{aj} - \frac{1}{3} \delta_{ij} r_a^2)$.

Parametrize 3-graviton vertex by  $\rightarrow (1+\beta_3) \times$ 

This will affect HHH $(1+\beta_3)$  and HHh $(1+\beta_3)$ 

$\Delta L_{cons} = -\beta_3 \frac{G_N^2 m_1 m_2 (m_1+m_2)}{r^2}$ $L_{rad} = \frac{1}{2M_{Pl}} [Q_{ij} R_{0i0j} + \cdots]$

$\frac{\dot{P}_b}{P_b} = -\frac{96}{5} G_N^{5/3} \nu M^{5/2} \left(\frac{P_b^\beta}{2\pi}\right)^{-8/3} [f(e) + \beta_3 g(e)]$ with correction from β_3. Compare $\dot{P}_b^\beta$ and $\dot{P}_b^{GR}$.

M and ν are modified by β_3. $\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$. $\dot{P}_b^\beta / \dot{P}^{GR} = 1 + \beta_3 g(e)$

where $g(e) \simeq 2.70$ for the Hulse-Binary pulsar. $\beta_3 = (4.8 \pm 7.8)\, 10^{-4}$, since $\dot{P}_b^{obs} / \dot{P}_b^{GR} = 1.0013(21) \simeq 1 + 2.70 \beta_3$. $\beta = 1 + \beta_3$ and $\gamma = 1$ for PPN parameters for the conservative part of the Lagrangian.

Perihelion of Mercury $\Rightarrow |\beta - 1| < 3 \cdot 10^{-3} \Rightarrow |\beta_3| < 3 \cdot 10^{-3}$, not so good

Lunar laser ranging: $|4\beta - \gamma - 3| < 9 \cdot 10^{-4} \Rightarrow |\beta_3| < 2 \cdot 10^{-4}$, of same order

β_3 modifies the orbital phase $\phi(t)$ already at 0PN level

$\phi^{0PN} = -(\Theta^{5/8} / \nu)(1 - 5\beta_3/2)$, $\Theta = \nu(t_c - t) M^{\cdots}$. $|\beta_3| < 3 \cdot 10^{-5} \left(\frac{M}{M_\odot}\right)\left(\frac{f_{min}}{10\,Hz}\right)$

if results will agree with GR to one orbit.

Final remarks: We have for the first time a measurement of the 3-graviton vertex β_3.

[Steve Carlip: PPN is just for conservative motion. Post-Keplerian expansion is another expansion, for strong fields, including radiation.] It has other parameters, such as for the time dependence of the Shapiro time delay.]

Valeri Frolov, "Black Holes and Hidden Symmetries"

P. Kanti, Int. J. Mod. Phys. A19: 4899-4951; Emparan & Reall; V.P. & Kubiz

Hidden symmetries play an important role in the study of 4 rotating black holes. They are responsible for separation of variable in the HJ, KG, and higher spin equations. Separation of variables in the Kerr metric is used for study of BH stability, particle and fie propagation, quasinormal modes, and Hawking radiation.

Brief history of 4D BHs: 1968 fourth integral of motion, separability of the HJ and KG equations in the Kerr ST, Carter's family of solut 1970 Walker and Penrose pointed out quadratic in momentum Carter's const is connected with a symmetric rank 2 Killing tensor. 1972 decoupling a separation of variables in EM and GP equations, massless neutrino cas massive Dirac case. 1973 Killing tensor is a 'square' of antisym. rank-2 Killing-Yano tensor (Penrose and Floyd). 1974: Integrability of ⇒ Type D. All Type-D without acceleration has KY tensor.

Main results (HD BHs): Kerr-NUT-(A)dS admit a principal conformal KY tensor, which generates a tower of Killing tensors.

$\xi_{(\mu;\nu)}=0$ (Killing eq). $\xi_{(\mu;\nu)}=g_{\mu\nu}\tilde{\xi}$ (conformal K. eq.).

CK: $K_{\mu_1\cdots\mu_n}=K_{(\mu_1\cdots\mu_n)}$, $\tilde{K}_{\mu_2\cdots\mu_n}\sim\nabla^{\mu_1}K_{\mu_1\cdots\mu_n}$, $K_{(\mu_1\cdots\mu_n;\nu)}=g_{\nu(\mu_1}\tilde{K}_{\mu_2\cdots\mu_n)}$.

CY: $k_{\mu_1\cdots\mu_n}=k_{[\mu_1\cdots\mu_n]}$, … $K_{\mu\nu}=f_{\mu\mu_2\cdots\mu_n}f_{\nu}{}^{\mu_2\cdots\mu_n}$ is a Killing tensor.

Principal conformal KY tensor: $\nabla_c h_{ab}=g_{ca}\xi_b-g_{cb}\xi_a$, ~~which~~ nondegenerate ⇒

⇒ $\nabla_{[a}h_{bc]}=0$, $\xi_a=\frac{1}{D-1}\nabla^n h_{na}$. $D=2n+\epsilon$, $h=db$. Properties: Hodge dual of CKY tensor is CKY tensor, of closed CKY is KY. External product of two closed CKY tensors is a closed CKY tenso

Darboux basis: $g_{ab}=\sum_\mu(e^\mu_a e^\mu_b+e^{\hat{\mu}}_a e^{\hat{\mu}}_b)+\epsilon e^{n+1}_a e^{n+1}_b$, $h_{ab}=\sum_\mu x_\mu e^\mu_a$ $m^\mu_\pm=\frac{1}{\sqrt{2}}(e^\mu\pm ie^{\hat{\mu}})$. $h\cdot m^\mu_\pm=\mp ix^\mu m^\mu_\pm$. A non-degenerate 2-form h ha n functionally independent eigenvalues. n essential coordinates x^μ and $n+\epsilon$ Killing coor ψ_i are used as canonical coordinates.

Killing-Yano tower: $h \Rightarrow h^{\wedge 2} = h\wedge h \Rightarrow \dots \Rightarrow h^{\wedge j} = h\wedge\dots\wedge h$.
$k_1 = *h$, $k_2 = *h^{\wedge 2}$, ..., $k_n = *h^{\wedge n}$. $K^1 = k_1\cdot k_1$, ... $K^j = k_j\cdot k_j$.
~~ξ~~. $\xi_a = \frac{1}{D-1}\nabla^n h_{na}$ is a primary Killing vector. $\nabla_{(a}\xi_{b)} = \frac{1}{D-2} R_{n(a}h_{b)}{}^{n}$
$\Rightarrow \xi_{(a;b)} = 0$, even off shell. Other KV's: $\xi_1 = K_1\cdot\xi$, ..., $\xi_{n-1} = K_{n-1}\cdot\xi$.
$n+\epsilon$ KV's, $n-1$ KT's, 1 g: $2n+\epsilon = D$ conserved quantities.
$\xi_i = \partial_{\psi_i}$, $d\psi_i = \sum_\mu \frac{(-x_\mu^2)^{n-1-i}}{U_\mu\sqrt{Q_\mu}} e^{\hat\mu}$, $U_\mu = \prod_{\nu\neq\mu}(x_\mu^2 - x_\nu^2)$.
Killing coords: $\psi_0, \psi_1, \dots, \psi_n$; essential coordinates x^μ.
A metric of a spacetime which admits a nondegenerate PCKY tensor can be written in the canonical form with functions $X_\mu(x^\mu)$: $e^\mu = \frac{1}{\sqrt{Q_\mu}}dx_\mu$, $e^{\hat\mu} = \sqrt{Q_\mu}\cdots d\psi_{\hat\mu}$. Obeying Einstein eq $\Rightarrow X_\mu = b_\mu x_\mu + \sum_{k=0}^{n} c_\mu^k x_\mu^{2k}$, the general Kerr-NUT-AdS metric.

Illustration: $ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$, $b = +\frac{1}{2}[-R^2dT + a(YdX - XdY)]$
$R^2 = X^2 + Y^2 + Z^2$. $h = db = dT\wedge(XdX + YdY + ZdZ) + a\,dY\wedge dX$,
$f = *h = X\,dZ\wedge dY + Z\,dY\wedge dX + Y\,dX\wedge dZ + a\,dZ\wedge dT$. $\xi_a^{(0)} = \frac{1}{3}h_{na}{}^{;n} = (-1,0,0,0)$,
$\xi^{(0)a}\partial_a = \partial_T$. $h_{TXX} = 1 = -g_{XX}\xi^{(0)}_T$, etc. $K = K_{ab}p^ap^b = \vec{L}^2 + 2aEL_z + a^2(E^2 - P_z^2)$.
$\xi^{(1)} = -K^{ab}\xi^{(0)}_b\partial_a = a^2\partial_T + a[Y\partial_X - X\partial_Y] = a^2\partial_T - a\partial_\varphi$. $t = T + a\varphi$, $\psi = -\varphi/a$,
$\partial_T = \partial_t$, $\partial_\varphi = a\partial_t - a^{-1}\partial_\psi$; $\xi^{(0)} = \partial_t$, $\xi^{(1)} = \partial_\psi$. $r^2 = -\Lambda_-$, $y^2 = \Lambda_+$ are essential coordinates, with $\Lambda_\pm = \frac{1}{2}[a^2 - R^2 \pm \sqrt{\qquad}\,]$. $X = a^{-1}\sqrt{(r^2+a^2)(a^2-y^2)}\cos\varphi$,
$Y = a^{-1}\sqrt{(r^2+a^2)(a^2-y^2)}\sin\varphi$, $Z = a^{-1}ry$, $b = \frac{1}{2}[(y^2 - r^2 - a^2)dt - r^2y^2d\psi]$
Most general solution of E. eq.: $R = (r^2+a^2)(1+\lambda r^2) - 2Mr$, $Y = (a^2-y^2)(1-\lambda y^2) + 2Ny$.
$r \to ix$, $M \to iM$ makes metric and potential b symmetric wrt $x \leftrightarrow y$.

Complete integrability of geodesic motion in general Kerr-NUT-AdS. Separability of HJ and KG eqs. Parallel transport along timelike geodesics: Page proved $F_{ab} = P_a^c P_b^d h_{cd}$ is parallel propagated along geodesics, so eigenvalues are constants of motion. Separability of massive Dirac eq. Stationary string eqs. are totally integrable. S eq. of grav. pert. Generalized KY tensors for minimally gauged sugra (5D EM with Chern-Simons).

Renaud Parentani, "Black Hole Radiation in Bose-Einstein Condensates"

I shall focus on one possible laboratory analogue of BH radiation. 0905.3634, 0903.2224. Motivations: 1. Compute the fluxes from "first principles" without using the analogy (Unruh '81) that the sound wave eq. in a non-homogeneous flow is analogous to the relativistic wav equation in curved spacetime. Because of dispersion, at short wavelen the (early) near horizon propagation is modified. Does this affect the properties of HR? Answer: When $K/\Lambda_{UV} \ll 1$, no, not significantly. Λ is the scale of dispersion, K is the surface gravity.

Conclusion: The analogy can be used to get a first approximation But first principles should be used to determine the validity of the analog

1st aim: Derive the phonon flux from the Bogolubov-de Gennes eq.

2nd aim: Explain the origin of the correlation pattern which has been "numerically observed" by Carusotto et al '08. Explain why it is not wash out by thermal noise, but reinforced.

3rd aim: Quantify the observable effects to guide experimenting BH radiatio

Last week a Technion group realized a BH-WH boson condensate.

Another critical frequency $\omega_{max} \neq \Lambda$ appears.

BEC - dilute gas. $\hat{H} = \frac{\hbar^2}{2m} \vec{\nabla}\hat{\psi} \cdot \vec{\nabla}\hat{\psi}^+ + V_{ext}|\hat{\psi}|^2 + g|\hat{\psi}|^4$

$\hat{\psi}(t,\vec{x})$ destroys an atom at $t, \vec{x}$. $V_{ext} = V_{ext}(t,\vec{x})$ and $g = g(t,x)$ can be tu

At low temperature, a large fraction of atoms condensates.

$\hat{\psi}(t,\vec{x}) = \psi_0(t,\vec{x}) + \hat{\chi}(t,\vec{x}) = \psi_0(t,x)(\hat{1} + \hat{\phi}(t,\vec{x}))$. Expand in ϕ. 0th ord gives GP eq. for ψ_0. 1st order gives Bog.-de Gennes eq.

$i\partial_t \psi_0 = (T + V + g\rho_0)\psi_0$, $\rho_0 = |\psi_0(t,\vec{x})|^2$ is Gross-Pitaevski eq (GP eq.)

Focus on static 1D condensate $\Rightarrow \psi_0 = e^{-i\omega t}\sqrt{\rho_0(x)}\, e^{iW(x)}$. $c^2(x) = g\rho_0/m$,

$v_0 = \frac{k_0}{m} = \frac{\partial_x W}{m}$. $i\partial_t \hat{\chi} = (T + V + 2g|\psi_0|^2)\hat{\chi} + g\psi_0^2\hat{\chi}^+$, the BdG eq., or

$i(\partial_t + v_0\partial_x)\hat{\phi} = T_\rho \hat{\phi} + mc^2(\hat{\phi} + \hat{\phi}^+)$, $T_\rho = -\frac{1}{2m\rho_0}\partial_x \rho_0 \partial_x$ (self-adjoint).

When gradients of c^2, ρ_0, v_0 are neglected (WKB), $(\omega - v_0 k_\omega(x))^2 = c^2 k_\omega^2 + \left(\frac{k_\omega^2}{2m}\right)^2$, a quartic dispersion relation. This fixes wave vector $k_\omega(x)$ and $\omega - v_0 k_\omega$.
Dispersionless limit: $m \to \infty$, $\left[(\omega + i v_0 \partial_x)\frac{1}{c^2}(\omega + i v_0 \partial_x) + v_0 \partial_x \frac{1}{v_0} \partial_x\right]\phi_\omega = 0$, not that of 2D relativistic field.

$$c(x) + v_0(x) = c_0 D\, th\left(\frac{\kappa x}{D c_0}\right) = 0 \quad (x=0) \text{ at the sonic horizon.}$$

$k(t) = k_0 e^{-i\kappa t}$, $x(t) = x_0 e^{\kappa t}$ for all ω ⇒ analogous to the near horizon propagation of a relativistic field. Say $c(x) = c_0 + (1-q)(c+v)(x)$, $v(x) = -c_0 + q(c+v)(x)$. $q=1$: only v varies (like BH). $q=0$: only c varies.
For $x \to \pm\infty$, $c, v \to c_\pm, v_\pm$ const ⇒ modes $e^{-i\omega t} e^{i k_\omega^\pm x}$ obeying $(\omega - v_\pm k_\omega)^2 = c_\pm^2 k_\omega^2 + \frac{k_\omega^4}{4m^2} = \Omega_\pm^2(k)$. Subsonic side: $c_+ > |v_+|$.
2 real sol: $k_+^u > 0$, $k_+^v < 0$. Supersonic side: $c_- < |v_-|$.
For $\omega > \omega_{max}$, as in the subsonic regime, 2 real & 2 complex.
For $\omega < \omega_{max}$, 4 real modes $\omega - vk = \pm\Omega = \pm\sqrt{c^2k^2 + \frac{k^4}{4m^2}}$.
ω_{max} cuts off phonon production: $\bar{N}(\omega) = 0$, $\omega > \omega_{max}$.
When $\omega > \omega_{max}$, 2-modes that mix elastically. When $\omega < \omega_{max}$: 3-mode sector. $\hat{\phi}_\omega = \hat{a}_\omega^{\dagger} \hat{\phi} + \cdots$ Enlarged Bog. transf. Unitarity is exactly preserved.
Number of Hawking particles is always less than the number of ~~particles~~ partners.
?[Frolov: When one considers scattering, the BH also has three modes]
For $\omega \ll \omega_{max}$, $T_{eff}(\omega) = $ const ⇒ const. T up to ω_{max}. For $\frac{\omega_{max}}{\kappa} \gtrsim 2$, $T_{eff}(\omega=0) = T_{Hawking} = \frac{\hbar\kappa}{2\pi}$ ⇒ extreme robustness of HR. $\frac{\omega_{max}}{\kappa} = 0.16 \Rightarrow T_{eff} = 0.6\, T_H$.

The problem is that in real BEC, the condensate temperature is never zero. $kT_0 \sim \frac{\hbar c}{\xi} \sim mc^2$, $\xi \doteq \frac{\hbar}{2mc}$ = "healing length." Bad new: $\partial_x(c+v)|_{hor} = \kappa < 1/\xi \Rightarrow T_{Hawking} < T_0$ "always" ⇒ Hawking radiation <u>hidden</u>?
In the in-vacuum $\bar{n}_\omega = |\beta_0|^2$. In a non-vacuum state, $\bar{n}_\omega = \bar{n}_\omega^{in} + |A_\omega|^2(\bar{n}_\omega^{v\,in} - \bar{n}_\omega^{in}) + |B_\omega|^2(1 + \bar{n}_\omega^{in} + n_\omega^{in}$
For $\frac{\omega}{\kappa} > 0.2$, $\bar{n}_\omega^{in}$ dominates/hides Hawking radiation ⇒ marginally possible detection.

Look instead at long distance correlations (instead of fluxes).
The correlations can be studied as simply as the fluxes.

Bill Unruh, "False Decoherence"

This is a warning that in some cases things can be deceptive

Interference - result of fixed phase between two alternatives.

$|\psi\rangle = |a\rangle + e^{i\phi}|b\rangle$. $\langle\psi|X|\psi\rangle = \langle a|X|a\rangle + \langle b|X|b\rangle + e^{i\phi}\langle a|X|b\rangle + e^{-i\phi}\langle b|X|a\rangle$

Correlations: $|\lambda\rangle|\psi\rangle \rightarrow |\phi\rangle = |\lambda_a\rangle|a\rangle + e^{i\phi}|\lambda_b\rangle|b\rangle$. $\langle\phi|X|\phi\rangle =$
$\langle a|X|a\rangle + \langle b|X|b\rangle + \langle\lambda_a|\lambda_b\rangle\langle a|X|b\rangle e^{i\phi} + \langle\lambda_b|\lambda_a\rangle\langle b|X|a\rangle e^{-i\phi}$.

If $\langle\lambda_a|\lambda_b\rangle = 0 \rightarrow$ no interference. This is called decoherence.

Once correlations have occurred → Interference is perm. destroyed

Under some circumstances decoherence "false" - does not destroy interfer

Zurek thinks this solves the measurement problem and gives classi probabilities. There is the view that once decoherence occurs, it is permanent. But sometimes the interference can be resurrected.

How do electrons interfere in an electron microscope? EM fields of electron at 2 positions do not "overlap." On the other hand, we know bloody well that electron microscope work, so there is interference.

H.O. $|\alpha\rangle = \frac{e^{\alpha a^+}}{e^{|\alpha|^2/2}}|0\rangle$, $\langle 0|\alpha\rangle = e^{-|\alpha|^2/2} = e^{-E/2\omega}$ with ratio of en to vacuum energy. EM field - take field of difference between two optic a dipole source. Write this as harmonic oscillator modes of the free f

Classical field has $E = \infty$ and so gives 0 overlap. $d = \sum \frac{E_i}{2\omega_i}$

Let me take a simpler model, a 2-level system connected to a 1-dimensional field. $\frac{1}{2}(\dot\phi^2 - \phi'^2 + m^2\phi^2) + \epsilon\dot\phi\sigma_3 h(x)dx$, $h(x)$ = slightly smeared δ function. $\rho(t) = \frac{1}{2}(1 + \vec{p}\cdot\vec{\sigma})$, $\vec{p} = Tr(\vec{\sigma}\rho) = Tr[\sigma T\{e^{iSHt}] + \cdots$

$\Phi(t,x) = \phi_0(t,x) + \sigma_3 \overset{\infty}{\epsilon} \int \frac{1}{2m} e^{-m|x-x'|} h(x')$, $\pi = \pi_0(t,x) + \epsilon(t)h(x)\sigma_3$, adial

i.e. $\sigma_3 = \pm$ displaces Φ field in various directions. Modes (H.O) are displaced in different directions by spin — overlap small.

If we look only at spin system, decoherence. But field tied to spin — Slow ($mt \gg 1$) changes drag field. Overlap collapses when ε returns to zero.

$\phi = \phi_0 + \frac{1}{2}\epsilon(t)\sigma_z e^{-m|x-x'|} h(x')dx'$ ($\epsilon(t)$ very slowly varying)

$\Delta\phi = m\epsilon(t)\int_{-k_c}^{k_c} \frac{e^{ikx}}{2\pi(k^2+(m-i\alpha)^2)}dk$, so ~~overlap~~ $S = \epsilon^2(t)\,O(1)$, overlap e goes as $O(1)\epsilon^2$ at end.

Start with pure state $\rho_x = 1$, $\rho_z(0) = 0$ $\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}$. Increase $\epsilon(t)$ slowly. State becomes maximally mixed when $\epsilon \neq 0$ but then pure again as $\epsilon \to 0$

Add $\frac{1}{2}H\sigma_y$ to Ham. (rotates spin). $\begin{pmatrix}1&0\\0&0\end{pmatrix} \to \begin{pmatrix}\frac{1}{2}&e^{-S}\\e^{-S}&\frac{1}{2}\end{pmatrix} \to \begin{pmatrix}0&0\\0&1\end{pmatrix}$.

Intermediate state decohered, but coherence returns [NOT Poincare recurrence] Field adiabatically dragged with system. This is really very different from spin echo, where spins rotate at different rates and then rotate back.

If in the middle I measured σ_x, ~~I would~~ rapidly, I would get 50-50. But if I measured it slowly, I would not get ~~interf~~ decoherence. So decoherence does not just depend on the system but on how one couples to the system. The field is tied to the oscillator, but only if one interacts rapidly with it. The same occurs for a deuteron if one measured the proton rapidly.

Penrose: Decoherence due to ~~non~~-overlap of grav. fields ⇒ destroys interference. Is this an example of false decoherence? It looks like the problem of the Coulomb field of an electron. But it is ~~more~~ subtle. I dont know. Source of decoherence is not directly due to non overlap of fields. Non-overlap creates a phase difference which leads to decoherence. $\frac{d}{dt}$(decoherence) = energy overlap (E). Decoherence = number overlap ($E/\hbar\omega$). The different gravitational fields cause a time uncertainty. A student and I are looking at spherical shells, to avoid gravitational radiation carrying off coherence. Penrose does not know what to expect. He says that if I dont get decoherence, perhaps gravity gives a new form of decoherence. But if classical GR gives no loss of coherence, the props for his argument are removed. [Sorkin: Penrose has other reasons.]

Harmonic oscillator with an ohmic heatbath.

$L = \frac{1}{2}\{\dot{q}^2 - \Omega^2 q^2 + \int(\dot{\phi}^2 - \phi'^2 + 2\epsilon q \dot{\phi}\delta)dx$. Solution

$\phi = \phi_0(t,x) - \frac{\epsilon}{2} q(t-|x|)$, $\ddot{q} + \frac{\epsilon^2}{2}\dot{q} + \Omega^2 q = \epsilon \dot{\phi}_0(t,0)$,

$\phi_0 = \phi_+(t+x) + \phi_-(t-x)$. Long time limit

$q(\omega) = \frac{\epsilon i\omega}{-\omega^2 + \frac{1}{2}\epsilon^2 i\omega + \Omega^2}(\phi_+(\omega) + \phi_-(\omega))$. $p = \dot{q} \Rightarrow p(\omega) = i\omega q(\omega)$.

Let us assume the heat bath ϕ is at zero temperature, so ϕ_0 is in the vacuum state. $\langle q^2\rangle = \int_0^\infty \langle q(-\omega)q(\omega)\rangle d\omega = \frac{1}{2\tilde{\Omega}}\left(1 - \frac{1}{\pi i}\ln\left(\frac{1+i\gamma/\tilde{\Omega}}{1-i\gamma/\tilde{\Omega}}\right)\right)$,

$\langle p^2\rangle = \frac{\Omega^2-\gamma^2}{2\tilde{\Omega}}\left(1 - \frac{1}{\pi i}\ln\left(\frac{1+i\gamma/\tilde{\Omega}}{1-i\gamma/\tilde{\Omega}}\right)\right) + \frac{2\gamma}{\pi}\ln\left(\frac{\Gamma^4}{\Omega^4}\right)$, logarithmically divergent.

$\rho = \exp\left[-\Lambda\left(\frac{1}{2}\left(\frac{p^2}{\alpha} + \alpha q^2\right)\right)\right]$, $\Lambda = \frac{\tilde{\Omega}}{T}$. $\langle p^2\rangle\langle q^2\rangle - \frac{\langle pq+qp\rangle^2}{4} = \frac{1}{2}\coth^2\left(\frac{\Lambda}{2}\right)$

A quantum oscillator tied to a heat bath at zero temperature has a reduced density matrix with nonzero temperature! The larger the coupling, the larger the temperature, though I don't know the frequency

$\tilde{\Omega} = \sqrt{\Omega^2 - \gamma^2}$, $\gamma = \frac{\epsilon^2}{4}$, Γ = cutoff.

State of oscillator is incoherent sum of "number" eigenstates. I now want to measure the temperature of the oscillator. Use second spring as a thermometer to measure temp. of first mass. What temperature does the thermometer measure? In the weak coupling limit, the temperature is zero. In the limit of weak coupling, the second oscillator is in its ground state, even though the first oscillator is in a thermal state. This is so even though heat bath—massless field—can carry away energy of arbitrary low frequency. If "temp" large, density matrix of first oscillator is incoherent.

False decoherence: Coupling between osc. and field creat correlations between osc. and field—adiabatically [thermometer has only long time scale effect on oscillator] dragged with oscillator. [Note—no mass gap—field does have arbitrarily low frequency modes.

On long timescales, correlations with field dragged with oscillator/system.

No good way of determining degrees of freedom.

If interaction rapid, degrees of freedom are "bare" degrees.
If interaction slow, degrees of freedom are "clothed" degrees.

Be careful!

Bei-Lok Hu, "Quantum Entanglement: Birth, Death, and Revival"

I'm going to finish my lecture from last year.

Entanglement is often regarded as *the* quantum property.

EPR and Quantum 'Nonlocality': Entanglement between Two Qubits/HOs interacting with a common EM field.

Common misconceptions:

1) Most people would say that the system of two qubits at rest is nonrelativistic, but the quantum field that interlinks them is intrinsically relativistic and determines their entanglement dynamics
2) They would say initial entanglement would stay there forever.
Not true for 2qb-env system, which is a quantum open system.
Start the system with an initial entangled state. Entanglement can dec
3) Entanglement dynamics behave very *differently* for different states.
4) Dependence of entanglement on the spatial separation of the two qubit

None of this refers to Q Nonlocality, which is a much deeper issue.

2 qubits interacting with the same EM field. Under Born-Mar approximation does not see distance dependence - Ficek and Tanas.

Pure states $|\psi_{AB}\rangle = |\phi_A\rangle \otimes |\phi_B\rangle$ separable, otherwise entangled.

$\rho(1,2) = \hat{\rho}(1) \otimes \hat{\rho}(2)$: simply separable or uncorrelated.

Wootters 1998 Concurrence, and logarithmic negativity are two measures

Yu, Eberly PRL 93, 140404 (2004) finite "sudden death" or finit time disentanglement for 2 qubits coupled to *separate* EM fields:

[Carlip: $\rho(1,2) = \rho(1) \otimes \rho(2)$ after a finite time] and remains there.

0801.0464: 2 HOs at same point in space interacting via a Q fie

Multi-mode and multi-qubit Jaynes-Cummings Hamiltonian:

Each qubit is separately coupled to the same EM field bath. Qubits do not couple to each other except through the field. Dipole and rotating wave appr

Take initial state $|0\rangle\otimes|\psi\rangle$, $|0\rangle$ = EMF vacuum, $|\psi\rangle$ = state of the two 2L atoms. $\rho^{ij}_{i'j'}(t) = \cdots$ = reduced density matrix of 2 atoms. Off-diagonal, non-Markovian F_t, G_t are trace-preserving linear operators on the space of density matrices. States 00, $+$, $-$, 11 with triplet $+ = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ and singlet $- = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Diagonal elements of the density matrix propagator decay exponentially. Off-diagonal terms evolve non-Markovian. Evolution equation for the reduced density matrix of the two qubits, while local-in-time, does not have constant-in-time coefficients. Hence, it does not correspond to a Markov master equation of the Lindblad type. (Although density matrix evolves locally in time, the correlations between different times are not given by the density matrix.) Under the non-Markovian evolution, for an initial Class A state we do not see sudden death of quantum entanglement and subsequent revivals, except when the qubits are sufficiently far apart.

2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space. A and B are HOs and ϕ is the scalar field. A and B are inertial, separation d.
$|\psi(0)\rangle = |q_L, q_R\rangle\otimes|0_M\rangle$.

Positive partial transpose (PPT) criterion [Peres 1996]:
$(Q_A, P_A; Q_B, P_B) \to (Q_A, P_A; Q_B, -P_B)$. If ρ^{PT} is a "good" Q state, ρ is separable.

"Degree of entanglement" vs. separation d oscillates in time.
At short distances, late time behavior tends to a steady state after transients.
$d_{ent} \approx \frac{1/\sqrt{2}}{\frac{2\gamma}{2} + \frac{\Lambda_1}{\pi/2}}$ = "entanglement distance." Initially entangled state has sudden death if $d \geq d_{ent}$, for an arbitrary two-mode squeezed state of A, B.

Don Marolf, "Observables and Information in Gravitational Physics"

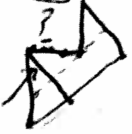
This is a huge subject. My two goals:

- Reconcile known gravitational physics with the suggested Unitarity of Black Hole Evaporation
- Explain AdS/CFT from a bulk perspective.

Main tool: H_{ADM} is a pure boundary term on shell.

Approach: Derive properties of classical GR which, if true in the quantum theory, would (largely?) achieve these goals.

It is plausible to me that there are QG theories with BH evaporation unitary and QG theories with it not unitary. I'm not going to give a QG theory.



Info "Paradox" in a Nutshell. Info is carried deep inside the black hole. Last null ray that misses strong curvature regio It will represent... Hawking radiation forms outside the BH. But string theory, AdS/CFT, BH pair production & other arguments suggest that this radiation should carry the info away when the BH evaporates. Can this tension be resolved? How could the info be transferred to the Hawking radiati

1) New non-locality or causality violation?

2) Infalling info is "stored" outside? I'd like to suggest this is the case.

Rough summary of claim: The grav. constraints relate operator that might a priori seem different. In particular, they allow info to b "stored" in an algebra of observables associated with both the Coulomb tail of the grav. field and propagating d.o.f. at infinity. Familiar example M is encoded in the grav. flux through i^0 [just Coulomb part].

Claim: With both parts, the algebra of distant observables encodes "all" info. From there, it can be transferred to the Hawking radiatio Precise statements will follow for various bdy conditions, using the fact that H_{ADM} is a pure boundary term (on shell).

Precise claims:

- Classical GR w/ AdS Boundary Conditions

Claim a) "AdS Boundary Unitarity": In the full theory, a cosmic censorship-like hypothesis implies that $A_{Bndy\,obs}(t_2) = A_{Bndy\,obs}(t_1)$, for all t_1, t_2. Corresponding properties in the QT would provide a mechanism for information to be "stored" outside the BH and transferred to the Hawking radiation. Conformally rescaled AdS acts much like a solid box in Minkowski spacetime.

In the classical theory, BHs form but do not evaporate. Nevertheless, the information is retained at the AdS boundary. The information is preserved at the boundary. [Bill Unruh: The evolution of the state could be non-unitary. It could be like an evolving damped harmonic oscillator.] I am just saying $q(t)$ and $p(t)$ gives $q(0)$ and $p(0)$. [~~Unruh:~~ I am claiming there is an invertible relation between the observables at one time and the observables at another time.

Claim b) "AdS Perturbative Holography": At any order $n \geq 1$ about a collapsing BH backgro of pert. theory (i.e., where Gauss' law is non-trivial), $A_{Pert\,obs}(\text{all } t) = A_{Pert\,obs}(t_1)$. $A_{Pert\,obs} = A_{Pert\,obs}$ (i^0 and early I^+: $u < u_0$). You just need an arbitrarily small part of scri-plus. This is true with massless matter. With massive matter, need things at future timelike ∞.

Background: AdS_D asymptotics and boundary observables

Use EOMs to expand fields in asymptotic series. 2nd order equations ⇒ 2nd order recursion ⇒ 2 independent pieces of data (up to gauge): "Dirichlet & Neumann." In Fefferman-Graham gauge w/ bdry @ $z = 0$:

$ds^2 = z^{-2}(dz^2 + g_{ij}(x,z)\,dx^i dx^j)$, $g_{ij} = g^{(0)}_{ij} + z\,g^{(1)}_{ij} + z^2 g^{(2)}_{ij} + \cdots + 16\pi G/(D-1)\,z^D T_{ij} + \cdots$

$g^{(n)}_{ij}$, $n > 0$, determined by $g^{(0)}_{ij}$ "Dirichlet." T_{ij} is independent of $g^{(0)}_{ij}$ "Neumann."

$T_{ij} = -E_{ij}(D-1)/8\pi G$, $E_{ij} = \lim_{z \to 0} z^{D-3} C_{ikjl} n^k n^l$ = electric part of the Weyl tensor.

if matter is present, it must fall off rapidly, but this is natural.

The equations rule out fractional powers or logarithms down to T_{ij} term. So asymptotic expansion tells a lot abo the solution, though there can be more than one solution with th same asymptotic data. Similar expansion for scalars in terms o ϕ_D and ϕ_N, with powers of z depending on m.

Which diffeos are gauge? Expect fall-off faster than isometries, i.e., vanish at Bndy of conformal compactification $\Rightarrow$ $g_{ij}^{(0)}$ is gauge invariant. Check: symplectic flux through Bndy, analogue of Klein-Gordon flux, $F(\delta g^1, \delta g^2)$ = flux out through Bnd

$= \int_{Bndy} [(\delta g^1)^{ij(0)} \delta T_{ij}^2 - (\delta g^2)^{ij(0)} \delta T_{ij}^1] \sqrt{-g^{(0)}}\, d^{D-1}x$.

Require BCs s.t. $F=0 \Rightarrow$ fix $g_{ij}^{(0)}$, even up to diffeos.

Similar for scalars. Say, fix ϕ_D. To preserve BCs, gauge transformations must fall off faster. Pick a point on the Any component of the FG expansion at x defines an observable. (FG fixes gauge for z, no reason to fix gauge for x.) $T_{ij}(x)$, $\phi_N(x)$ are "Boundary Observables." If $g_{ij}^{(0)}(x)$ is time dependent, the energy is time dependent, so one gets a time-depend Hamiltonian. $g_{ij}(x)$ is a boundary condition that can be imposed. Different $g_{ij}^{(0)}(x)$ give different phase spaces classically or differen Hilbert spaces quantum mechanically. [Sorkin: AdS is not globally hyperbo Boundary condition $g_{ij}^{(0)}(x)$ controls the information that can come in fro

Ia "Boundary Unitarity" Simple case: Assume $g_{ij}^{(0)}$ has time-tran

1. "Boundary Fields" form a natural set of <u>observables</u>. Let $A_{bndy\,obs}(t)$ = Poisson algebra of bndy observables [generated by ϕ_N, T_{ij} at time t.
2. Construct the Hamiltonian: On the constraint surface, H is a pure bndy t (Time-dependent of t-trans not a symmetry.) $H = H(t) \in A_{bndy\,obs}(t)$. [weak equivalence, or action on physical phase space.]

$H(t) = \int_{\text{Bndy Cut w/ } t = \text{const}} T_{ij}\, \xi^i\, \underline{n}^i\, dA$ with $\xi = \partial_t = \underline{n}^i$ = normal to t = constant cut of boundary. Note: For any observable $\mathcal{O}$, $\partial_t \mathcal{O}(t) = -i\left[\mathcal{O}(t), H\right]$.

3. Suppose* that we can exponentiate H to define $e^{iH\Delta t}$ as an operator on boundary observables that preserves the Poisson structure. Then, $\mathcal{O}(t_2) = e^{-iH\Delta t}\,\mathcal{O}(t_1)\,e^{iH\Delta t}$. $\mathcal{O}(t)$ is any gauge-invariant function on the phase space. Since it is gauge invariant, it doesn't matter how one extends t into the interior. I.e., any Bndy Obs at t_2 can be expressed in terms of Bndy Fields ϕ_N, T_{ij} at any other t_1.

$\Rightarrow A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$, "Boundary Unitarity!"

In QM, information present on the Bndy at any one time t_1 remains present at any other time t_2.

Suppose* that we can exponentiate $H(t)$ to define $U(t_1, t_2) = T\exp(-i\int_{t_1}^{t_2} H(t)\,dt)$. Classical interpretation on space of smooth metrics: Assumes long-time existence of solutions to EOMs, at least in some neighborhood of the Bndy. I.e., form of "Cosmic Censorship" is needed to allow the Hamiltonian to be exponentiated.

QM interpretation: Assumes quantum Hamiltonian can still be built from ϕ_N, T_{ij}, but QG resolves classical violations of cosmic censorship.

A "physical" example with superobserver outside AdS. He throws in red or blue particles into an ensemble. Suppose that the Green "super-observer" has access to an ensemble of AdS spaces, all prepared the same way. [I.e., first observer makes the same choice for each.] To find out what was thrown in earlier, at time t_2 the Green observer:

1. Carefully measures the energy of each system.
2. Looks for red or blue particles near infinity at time t_2 $\langle O(t_2)\rangle$
3. Carefully measures each energy again, and
4. Performs a certain interference experiment.

Use this to compute (for all E, λ, E'): $f(E,\lambda,E') = \langle\psi|P_{H=E}P_{O(t_2)=\lambda}P_{H=}$

Now compute the prob. distribution ~~for~~ for $O(t_1)$ via

$$\langle\psi|P_{O(t_1)=\lambda}|\psi\rangle = \langle\psi|e^{-iH(t_1-t_2)}P_{O(t_2)=\lambda}e^{iH(t_1-t_2)}|\psi\rangle$$
$$= \int dE\,dE'\, f(E,\lambda,E')\,e^{-i(E-E')(t_1-t_2)}$$

The red and blue particles have different commutators with the Hamiltonian. It is not that the red and blue particl have different energies.

It is very difficult to write down local observables in quantum gravity. One can extend t arbitrarily insi

Summary:

1. At the classical level, Perturbative Holography & (for AdS) Bndy Unitarity follow from grav. constraints, gauge invarian and a form of Cosmic Censorship.
2. Info is stored in asymptotic local observables.
3. If these properties also hold in the quantum theory, info can be transf to Hawking radiation via constraints and local energy conservatio & no causality violation or non-locality required

II. Classical GR w/ As. Flat Boundary Conditions

"As. Flat Perturbative Holography!" At any order $n \geq 1$ of pert. theory, $A_{\text{Pert,bdy}} = A_{\text{Pert,bulk}}$ (i^0 and early I^+: $u < u_0$) about a collapsing black hole background. Again, a correspon property in the quantum theory would provide a mechanism f information to be "stored" outside the black hole and transferred to the Hawking radiation

Rafael Sorkin, "What Is a Quantal Reality?"

Title: "Logic : The Quantum :: Geometry : Gravity"

1. Quantum Gravity and Quantal Reality
2. The main inputs: histories, preclusion, anhomomorphic coevents

a. histories b. preclusion and the q-measure c. The 3-slit experimen

d. anhomomorphic coevents: logical inference as dynamics

3. The multiplicative scheme
4. Preclusive separability and "the measurement problem"
5. Illustration: An EPRB experiment
6. open questions

QG and QR. We still haven't learned how to think clearly about the quantum world in itself, without reference to "observers" and other external agents. Because of this we don't really know how to think about the Planckian regime where quantum gravity is expected to be most relevant. Without an observer-free notion of reality, how does one give meaning to superluminal causation or its absence in a causal set? We all employ intuitive pictures in our work, but we lack a coherent descriptive framework to answer: What is a quantal reality? My main purposes are to - propose a (family of possible) answers

- show how the measurement problem can be solved.

Histories (the kinematic input)

In the classical era it was easy to say what a possible reality was. Examples: GR (a 4-geometry), Brownian motion (a single worldline). We could survey all the possible realities, and state the dynamical laws that further subscribed them (i.e., e.o.m. or field eqs). Ω = space of all histories. Event = subset of Ω (e.g. "It rained all day yesterday". An event should be a measurable subset of Ω. I shall assume Ω is finite

Coevent = ϕ (defined here for future reference) (It answers every possible question of the form "Did this event happen?" "Will that event happen?") It is a map from events $\mathcal{Q}$ to Z_2 (yes or no). (It's higher order: a "predicate of predicates") Classically "existence" corresponds to a single history.

$\mathcal{Q} = 2^{\Omega}$ = set of all events. ϕ maps $\mathcal{Q}$ to Z_2.

$|\Omega| = N$, $|\mathcal{Q}| = 2^N$, $|\mathcal{Q}^*| = 2^{2^N}$. $\mathcal{Q}^*$ = set of coevents.

Quantally "existence" ought to be a quantal history, but what exactly should this mean? (It will not be a wave function Schrodinger eq. won't enter the story.) Idea is to use a path history, but not to get a wavefunction.

Preclusion and the Q-measure (dynamical input)

Anhomomorphic logic grows out of the path-integral. What does the path-integral really compute? The probability of a succession of "position events" can be written directly as a ~~path~~ integral. $A \subseteq \Omega$, $A \in \mathcal{Q}$, $\mu(A) \geq 0$.

The resulting expression makes sense for any event (set of histori

This q-measure μ or "decoherence functional" is what the p.i. computes! Mathematically can view QM as level two measure theor (ongoing test at IQC). μ can't be interpreted as a probability in general because of interference. In general we dont know what μ means. (From histories standpoint, this is the problem of quantum interp.) I propose to interpret μ in terms of a Preclusion Postulate. μ = diagonal element of decoherence functiona

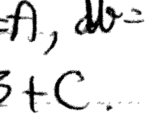
Classically, for A and B disjoint, and A + B = union of A and B

$\mu(A+B) = \mu(A) + \mu(B)$. But quantally, $\mu(A+B) \neq \mu(A) + \mu(B)$.

However, there is no new interference for triplets, because μ = |amplitud

$\mu(A+B+C)-\mu(A+B)-\mu(A+C)-\mu(B+C)+\mu(A)+\mu(B)+\mu(C)=0$ in QM. This is being tested at the Institute of Quantum Computing at Perimeter. At first they got a 10σ violation, but no one believes this.

Preclusion Postulate: $\mu(E)=0 \Rightarrow E$ does not happen $[\phi(E)=0]$. Call such a coevent ϕ preclusive (cf. Cournot's principle). The idea is that the whole dynamical content of the quantal formalism reduces to this preclusion rule (with approx. preclusion if need be). Cournot's principle is that the ultimate meaning of classical probability is this preclusion, which unfortunately needs to be approximate.

The 3-slit paradox: 

Events da, db, dc, da+db, ..., d. Write da=A, db=B, dc=C, then d=A+B+C. preclusions A+B, B+C.
$\mu(d)=\mu(A+B+C)>0$ but $\mu(A+B)=0=\mu(B+C)$. These are all intrinsic events, not "measured events." Classically: A+B and B+C cannot happen, but A+B+C can! The whole (partial) history space is covered by precluded events (compare on histories view: K-S, Stairs' version thereof, GHZ, Hardy). Logical contradiction, as whole history space is covered by excluded events. These logical paradoxes are a good starting point.

Anhomomorphic coevents: Let us retain preclusion unchanged. Logical inference (deduction) is special case of dynamics (Kepler's laws to forecast eclipses) (logic concerns events, not strings of words). Logic has been "ossified" like geometry was. Should bring it into physics. The logical triad: $\phi: \mathfrak{A} \to Z_2$. $\mathfrak{A}$ holds the "questions," ϕ answers them.

Each (dynamically allowed) ϕ describes a possible reality: a "possible quantal history." Rules of logical inference are conditions on ϕ. We will preserve $\mathbb{Q}$ and Z_2 but modify these conditions so as to accommodate interference (overlapping preclusions). Both truth and falsehood matter here. Affirming the particle is here differs from denying the particle is elsewhere (cf. tetralemma).

What are the classical rules of inference?

(1a) modus ponens $[\phi(A)=\phi(A\to B)=1 \Rightarrow \phi(B)=1]$ (1b) $\phi(A)=0\Rightarrow\phi(\neg A$ (1c) $\phi(\emptyset)=0$.
$A\to B=(\neg A)\cup B$ (material implication)

(2) ϕ is a homomorphism of unital Boolean algebras (ϕ preserves $+$ and $\times$ and 1, or equivalently, & and $\neg$)

(3) $\phi^{-1}(1)$ is a maximal preclusive filter in Ω ← (the set of all events that happen)

(1), (2), and (3) are equivalent if the pattern of preclusion is classical.

These rules imply that ϕ may be characterized by a single history.

The Multiplicative Scheme (as an example)

We retain condition (3) word for word as the definition of Primitive Preclusive Coevent. (1a) survives but (1b) does not. (2) survives in part: ϕ preserves &.

Equivalent formulation: Every multiplicative ϕ has a "support" F in Ω st $\phi=F^*$. The coevent ϕ affirms the event A iff $F\subseteq A$. ϕ is primitive when F is as small as possible while remaining preclusive. (Truth/happening is a "collective property" of histories.) This easily reduces to classical logic when interference is absent (also classical deterministic theories, which don't even have μ!)

Illustrate this scheme with 3-slit. Two preclusive coevents: $(A+C)^*$, $(A+B+C)^*$. One primitive coevent: $(A+C)^* = A^*C^*$.

Two events happen: $A+C$, $A+B+C$, six do not (e.g. A, $A+B$).

The event $\Omega = A+B+C$ can happen, so the paradox is removed.

The proposal is that a primitive coevent corresponds to reality.

Preclusive Separability and "The Measurement Problem"

Coevents describe microscopic reality directly and anhomomorphic inference resolves the logical paradoxes of qm.
We hope it will light the way to QSC for posets.

Thm (in the Multiplicative Event):

Let ϕ be a PPC and let $\Omega = \Omega' + \Omega''$ be a partition such that A is precluded iff its intersections with Ω' and Ω'' are both precluded. Then support (ϕ) lies within either Ω' or Ω''.
Therefore either Ω' or Ω'' happens, but not both.
But are macroscopic events preclusively separable in this way?
A sufficient condition: No event in Ω' interferes with any event in Ω'' (a very strong type of "decoherence", closely related to idea of a record). The following weaker condition suffices and I think is plausible: If a subevent A of Ω' lies within an precluded event B, then it lies within a precluded subevent C of Ω'. This then reduces to a calculation.

Steve Carlip, "Spontaneous Dimensional Reduction in Short-Distance Quantum Gravi

I've always thought of Peyresq as a place to give crazy talks. The results are very preliminary. Some in the audience may know more

Accumulating bits of evidence that quantum gravity simplifies at short distances, a dimensional reduction to 2 dimensions. The simplest evidence is from causal dynamical triangulations. Approximate path integral by sum over discrete triangulated manifolds. $\int[dg]e^{iI_{EH}[g]} \Rightarrow \sum e^{iI_{Regge}[\Delta]}$. It is Euclidean but in a more Lorentzi way than usual.

Old results: two phases: (1) "crumpled": Hausdorff dimension $\gg 1$ (2) "branched polymer": dimension ~ 1. New ingredient (Ambjorn, Jurkiewicz, Loll) in some sense made the path integral more Lorentzian: Start with a time slicing $R \times$ 3-manifold. Then one gets a reasonably straightforward analytic continuation for each contribution of the path integral as one changes a timelike leg from Euclidean to Lorentzi

(1) Fixed time slicing. (2) No topology change/baby universes.

How do you "measure" the dimension of spacetime?

Spectral dimension d_S: dimension of spacetime seen by random walker. Heat kernel $K(x,x';s)$: $\left(\frac{\partial}{\partial s}-\Delta_x\right)K(x,x';s)=0$. At ~~short distance~~ small s,

$K(x,x';s)\sim(4\pi s)^{-d_s/2}e^{-\sigma(x,x')/2s}(1+\cdots)$, $K(x,x;s)\sim(4\pi s)^{-d_s/2}$

Ambjorn, Jurkiewicz, and Loll: $d_S(\sigma\to\infty)=4$, $d_S(\sigma\to 0)\approx 2$.

Propagator $G(x,x')\sim\int_0^\infty ds\, K(x,x';s)\sim\begin{cases}\sigma^{-1}(x,x') & \sigma \text{ large}\\ \log|\sigma(x,x')| & \sigma \text{ small}\end{cases}$

Short distances: characteristic behavior of a propagator in two dimension

Crossover happens a bit above the bare Planck scale.

One has a topological distinction between space and time, a distinctio between timelike and spacelike legs. The positive cosmological constan gives convergence. It is not believed to work for zero or negative Λ.

The heat kernel is calculated by random walks, with s being the number of steps. You fix the total starting number of simplices You start with a random configuration. You do moves that change the configurations randomly and then do a Monte Carlo calculation. The moves can change the number of simplices, and $\Lambda > 0$ damps larger number of simplices.

Renormalization group analysis of GR gives the second piece of evidence. This goes back to the old idea of asymptotic safety of Weinberg. It is possible there is a UV fixed point. Even though the theory may be nonrenormalizable, one can still make an infinite number of predictions. One may even have the infinite number of coupling constants controlled by a finite number of constants.

Lauscher, Reuter, Niedermaier, etc.: Look at renormalization group flow for Einstein gravity plus higher derivative terms.
- Define scale-dependent effective action ... Find at UV fixed point 2-dimensional behavior.

General argument (Niedermaier): $\frac{1}{16\pi G_N} = \frac{\mu^{d-2}}{g_N(\mu)}$.
RG flow: $\mu \frac{\partial}{\partial \mu} g_N = [d-2+\eta_N(g_N)] g_N$ with anom. dim η_N.
If there is a (finite) non-Gaussian fixed point $g_N^* > 0$, then $\eta_N(g_N^*) = 2-d$. But propagator $\sim (p^2)^{-1+\eta_N/2} \sim (p^2)^{-d/2} \Rightarrow$ logarithmic behavior in position space. [Don Marolf: This is just the statement that if one had a classically scale-independent theory of gravity, it must be 2D; only there is the gravitational coupling constant dimensionless.] [Valeri Frolov: You can also get this behavior of the propagator in 4D from higher-derivative terms.] Because the action is truncated to quadratic order with pure gravity, most people are sceptical of the results.

Loop quantum gravity area spectrum $A \sim \sqrt{l_j^2(l_j^2 + l_p^2)}$ with $l_j = \sqrt{j}\, l_p$, (j = integer or half integer) is another piece of evidence. For large areas, $A \sim l_j^2$. For small areas, $A \sim l_j l_p$. Modesto: assume metric scales as areas under $l \to \lambda l$; then reproduce causal dynamical triangulation result for spectral dimension. I'm not sure whether to take any of this seriously.

Anisotropic scaling models give another piece of evidence. Hořava: new class of renormalizable (but not diffeo-invariant) models of gravity: - short distances, nonrelativistic gravitons (broken Lorentz symmetry) - large distances: approaches Einstein gravity. Anisotropic scaling ($x \to bx$, $t \to b^3 t$). Then diffusion described by operator $\sim \Delta^2 \Rightarrow d_S = 2$ at short distances.

Short distance approximation. Wheeler-DeWitt equation:

$$\left\{16\pi l_p^2 G_{ijkl} \frac{\delta}{\delta g_{ij}} \frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi l_p^2}\sqrt{g}\, {}^3R\right\}\Psi[g] = 0.$$

"strong coupling" ($G \to \infty$) $\Leftrightarrow$ "small distance" ($l_p \to \infty$) $\Leftrightarrow$ "ultralocal" (no spatial derivatives). Classical solution: - Kasner at each point if $l_p \to \infty$ - normally BKL/Mixmaster if l_p large but finite (Kasner eras with bounces in which axes change).

Any signs of "dimensional reduction"? Which dimension is picked out? I think the answer is yes. Geodesics in Kasner $ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2$ with $-\frac{1}{3} < p_1 < 0 < p_2 < p_3$, $p_1 + p_2 + p_3 = 1 = p_1^2 + p_2^2 + p_3^2$. Small t: $\Delta_x = t^{p_1}x \sim t^{p_1}$, $\Delta_y = t^{p_2}y \sim 0$, $\Delta_z = t^{p_3}z \sim 0$. Large t: $\Delta_x = t^{p_1}x \sim t$, $\Delta_y = t^{p_2}y \sim t^{\max(p_2, 1+p_1-p_2)}$, $\Delta_z = t^{p_3}z \sim t^{p_3}$. Geodesics explore a nearly one-dimensional space. Particle horizon approaches line as $t \to 0$

Heat kernel for Kasner space. People looked and failed. Various approximations (Futamase, Berkin): $K(x,x;s) \sim \frac{1}{(4\pi s)^2}(1 + Qs)$, $Q \sim \frac{1}{t^2}$. Small t: Q term dominates, $d_S \sim 2$. HaMiDeW coeffs: $K \sim \frac{1}{(4\pi s)^2}(1 + [a_1]s + [a_2]s^2 + \ldots$ $[a_1]$ gives $\ln \sigma(x, x')$ term in propagator, when you put in almost arbitrary matter.

Asymptotic silence? Cosmology near generic spacelike singularity: - Asymptotic silence: light cones shrink to timelike lines. - Asymptotic locality: inhomogeneities fall outside shrinking horizons faster than they grow ⇒ spatial points decouple; BKL behavior. Underlying physics: extreme focusing near initial singularity. Is this also true at very short distances?

Raychaudhuri equation: $\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_\alpha{}^\beta\sigma_\beta{}^\alpha + \omega_{\alpha\beta}\omega^{\alpha\beta} - R_{\alpha\beta}k^\alpha k^\beta$.
Quantum fluctuations: $\langle\theta^2\rangle = \langle\theta\rangle^2 + (\Delta\theta)^2$, $\langle\sigma^2\rangle = \langle\sigma\rangle^2 + (\Delta\sigma)^2$.
Fluctuations focus null geodesics: large effect at Planck scale?
Note that θ is conjugate to area A: $\Delta\theta\Delta A \sim l_p$. $\Delta A \sim l_p^2 \Rightarrow \Delta\theta \sim 1/l_p$.
Some further hints from renormalization group analysis: $R \sim l_p^{-2}$ near Planck scale. Does spacetime foam focus geodesics?
[Larry Ford: $R_{\alpha\beta}k^\alpha k^\beta$ can have negative quantum fluctuations, which might cancel the focusing from $(\Delta\theta)^2$ and $(\Delta\sigma)^2$.]

Short-distance picture:
- short distance asymptotic silence
- "random" direction at each point in space
  - not changing too rapidly in space: regions of size $\gtrsim l_p$ fairly independent
  - evolving in time; "bouncing," axes rotating, etc.
- effective two-dimensional behavior: dynamics concentrated along preferred directi
- space threaded by lines; spacetime foliated by not-very-smooth two-surfac
- product wave function "$\prod_x \Psi[\alpha,\beta]$"

Can we use this?
't Hooft, Verlinde and Verlinde, Kabat and Ortiz: eikonal approximation $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + h_{ij}dy^i dy^j$ with different natural scales for the two metrics.

Ralf Schuetzhold, "On the Scale(s) of Quantum Gravity"

For many problems, asking the right questions is a big step towards the solution. E.g., when (at which scale) does QG become important? Classical gravity for 1kg, 1g, 1mg, 1μg, 1ng ...? (virus: 10ag, bacterium: 10pg). Lightest neutrino $0 \overset{?}{<} m_\nu < 2.2\,eV$, $J = \hbar/2$, $Q = 0$. Kerr solution: singularity at $a = J/m_\nu \geq O(1\mu m)$!? Planck mass $m_P \approx 22\mu g$ (e.g., grain of dust) → question: classical gravity for neutrino/dust grain? $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa\langle\hat{T}_{\mu\nu}\rangle_{ren}$ or $\kappa\hat{T}_{\mu\nu}$??? Further examples: Planck momentum $p_P \approx 6.5\,kg\,m/s$. Planck force $F_P \approx 1.2 \cdot 10^{44}\,N$

Resolution? Question: classical gravity for neutrino/dust grai Remember question from QM: Does the electron take the left or right slit or both? - ill-posed question → duality (wave-particle). Same in QG? - measurement accuracy (undecidable). $\hat{g}^{\mu\nu} = g^{\mu\nu}_{class} + \delta\hat{g}^{\mu\nu}$. Analogy to Bose-Einstein condensates $\hat{\rho} = \hat{\Psi}^\dagger\hat{\Psi} = \rho_{class} + \delta\hat{\rho}$. Shot noise ($N < \infty$) + quantum fluctuations ($g > 0$): $\hat{\Psi} = \psi_{condensate} + \hat{\chi}$.

Black Hole Info "Paradox" Question: does the information escape? Short reminder: · BH of mass M_{BH} · Hawking $T_H \propto M_P^3/M_{BH}$ · entropy $S \propto M_{BH}^2/M$ → information · number of Hawking photons $N \propto M_{BH}^3/M_P^3$ → $O(1)$ bit per emitted photon (maximum). Well-posed question? → measurement accura → origin of Hawking radiation...

Bill Unruh's idea: Sound waves in irrotational flow $\delta v = \nabla\phi$
$$\left(\frac{\partial}{\partial t} + \nabla\cdot v_0\right)\frac{\rho_0}{c_s^2}\left(\frac{\partial}{\partial t} + v_0\cdot\nabla\right)\phi = \nabla\cdot(\rho_0\nabla\phi).$$
Scalar field ϕ in curved space-time $\Box_{eff}\,\phi = 0$ in $g^{\mu\nu} = \ldots$. De Laval nozzle makes a BH with $T_H = \frac{\hbar}{2\pi k_B}\left|\frac{\partial}{\partial r}(v_0 - c_s)\right| = O(nK \ldots K)$. Toy model for underlying theory (quantum gravity?) → trans-Planckian problem. Experimentally measurable?! Origin of Hawking radiation: W. G. Unruh and R.S., Phys. Rev. D 71, 024028 (2005). Sub-luminal (e.g., He) vs. super-luminal (BEC).

Supersonic dispersion: Initial wave-packet in its ground state is ripped apart into: Hawking radiation plus infalling partner. Entanglement (squeezed state) → thermality. Creation near horizon at low energies. Encoding information??

Eddington-Finkelstein metric with changed dispersion relation

Lessons for QG? Hawking radiation is quite robust (i.e., independent of the microscopic structure) for a large class of systems and does not require the Einstein equations. But there are also (physical) examples, which show deviations from the Hawking effect. ~~Impact~~ Hard to put in info without greatly changing Hawking spectrum.

Teleportation: Problem: Alice has $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow$ Bob. They share a maximally entangled state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. Hawking radiation $|\psi\rangle \approx \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$. Wild speculation: Teleportation from the BH interior. Alice: inside $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow$ Bob: outsi $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{Hawk}|0\rangle_{infall} + |1\rangle_{Hawk}|1\rangle_{infall})$. Alice measurement → classical gravitational field. Fluid analogue: classical density / pressure. If Alice were inside the BH, how can she send information outside? If she can change the ADM mass, Bob can measure this outside.

Resolution? Question: does the information escape? Maybe again ill-posed question? · teleportation facilitates escape in principle, but - grav field cannot be measured accurately enough to retrieve information $\hat{g}^{\mu\nu} = g^{\mu\nu}_{class} + \delta\hat{g}^{\mu\nu}$. Measurement accuracy → Heisenberg uncertainty. Both points of view are equally correct/wrong?

Cosmological constant. Kinematics of phonons: quantitative analogy $g^{\mu\nu}_{eff}$. Dynamics of background: only qualitative analogy. E.g., Bose-Einstein condensates.

Lessons for QG? - we can quantize (linearized) phonons for small k. - beyond linear order: UV divergences (non-ren.)

- sum of zero-point fluctuations of phonon modes (extrapolating Euler eq. to large k) up to cut-off does not yield correct result [R.S., Proceedings of Science (QG-Ph) 036 (2007)].
- quantization of vorticity from Euler equation??
  - Bose-Einstein condensates $\xi \Leftrightarrow m \Leftrightarrow \Gamma$
  - superfluid Helium $\xi_{roton} \not\Leftrightarrow m \Leftrightarrow \Gamma$
- several cut-off scales ($\rightarrow$ Planck scale?)
  - breakdown of Euler $\omega^2 = c^2 k^2 (1 \pm k^2 \xi^2)$
  - circulation quantum Γ of one vortex
  - UV cut-off $\ll \xi$

$\int d^3k \, k \sim \Lambda^4_{cutoff}$ is just wrong, with Λ_{cutoff} = healing length

[Parentani: The contribution to the pressure does not agre with the naive calculation of free modes, even at very low k.

Summary: Many questions:

- classical gravity for dust grain (vs. neutrino)
- w

Luca Bombelli, "Lorentzian Manifolds and Causal Sets as Ordered Measure spaces

Lagrangian-based approach to dynamics of causal sets and matter fields

Causal set: A partially ordered set $(S,<)$ ∋ $\forall x,y,z \in S$, $x<y$ and $y<z \Rightarrow x<z$. $\forall x\in S$, $x \not< x$

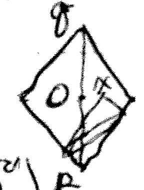
Of course, for any $R \subset S$ $\mu(R)=|R| \Rightarrow$ causal measure space.

Lorentzian geometries (M,g) distinguishing $(M,\hat{g},|g|)$, $(M\cancel{z},<,|g|)$ ... which satisfies $\forall x,y$ if $A(x,y)=\{z \mid x<z<y\}$ then $|A(x,y)|<\infty$.
→ causal measure spaces $(X,<,\mu)$

Kinematics. Conjecture: If $(S,<)$ is manifoldlike then (M,g) almost unique.  $V=\frac{\pi}{24}\tau^4+(AR_{00}+BR)\tau^6$. What do we mean by a manifoldlike causal set? For large N, very few posets on N elements look anything like manifolds. $\exists S' \to (M,g)$. s.t. $i(S)$ uniform with density ρ. $x<y \Leftrightarrow x\in I^-(y)$. Also want the manifold to have negligible structure on the scale of the separation between elements of the causal set. Conjecture is for large but finite N

Dynamics? How do we go about doing dynamics? We could try to mimic $S_{EH}=\frac{1}{16\pi G}\int R\,dX$ by $\frac{1}{16\pi G}\sum_{x\in S}\rho^{-1}$"$R$" but need to say what "$R$" is. Causality–curvature relation: In 4D, 
$V=\frac{\pi}{24}\tau^4+\frac{\pi}{760}(R_{00}+\frac{1}{6}R)\tau^6+O(\tau^8)=V(A)$. The problem is the pesky R_{00}.

Rafael Sorkin and David Rideout had a different approach, building up causal sets sequentially.



In d dim, $V(A)=k_d\tau^d[1+(A_dR+B_dR_{00})\tau^2+\cdots]$

$\frac{V(A)}{k_d\tau^d}-1=(A_dR+B_dR_{00})\tau^2=(A_dR_{\mu\nu}g^{\mu\nu}\tau^2+B_dR_{\mu\nu}(q-p)^\mu(q-p)^\nu)$

Integrate over the inside with q replaced by the point.
Find 2 linear equations for R & R_{00}. In 4D, the solution is
$R=\frac{162}{\pi\tau^2}\left(\frac{24V(p,q)}{\pi\tau^4(p,q)}-1\right)-\frac{12960}{\pi\tau^6}\int_x\left(\frac{24V(p,x)}{\pi\tau^4(p,x)}-1\right)d^4x$. This depends on p and q.

However, if T is the length of the chain, the time from bottom to ~~top~~ $\tau = K_d\langle T\rangle$, with some dimension-dependent proportionality. Also, you would need the dimension.

Lagrangian generator $K_E(p,q,r) \begin{cases} \frac{1}{16\pi G} & p<r<q \\ 0 & \text{otherwise} \end{cases}$.

Pre-Lagrangian $\mathcal{L}_E(p,q) = \cancel{V^3(A(p,q))} \int_x \frac{1}{16\pi G}\left[V(A(p,q))^2 + E\int dN_0 Y(A(p,\Delta))\right]$.

In $d=4$, $E = \frac{12960}{\pi}$. I know E when the causal set is manifold-like. E must be the coefficient that makes the result invariant under boost

Conclusion: Maybe I'll just quit here.

Diego Blas, "On the Extra Mode and Inconsistency of Horava Gravity"
0906.3046 with O. Pujolas(CERN) and S. Sibiryaka(EPFL)

The proposal is to enhance the UV behavior of gravity by $P \sim \frac{1}{\omega^2 - c^2 k^2 - \alpha (k^2)^z}$ with high enough power z to make loop integrals finite, $\int d^4k\, k^2 P$. For low energy, $\omega^2 - c^2k^2$ dominates; in UV $-\alpha(k^2)^z$ dominates.

i) Lorentz invariance is violated. To give LI at low energy requires fine tuning, but that is OK if we have a renormalizable theory.

ii) ~~LI~~ $\xrightarrow{?}$ ~~Diff~~

Horava says spacetime has a preferred slicing, a preferred time, with γ_{ij}, N_i, N. This leaves the gauge invariance Diff $\to$ $x^i \mapsto \tilde{x}^i(x^i, t)$ and $t \mapsto \tilde{t}(t)$.

$$ds^2 = (N^2 - N_iN^i)dt^2 - 2N_i\, dt\, dx^i - \gamma_{ij}dx^i dx^j.$$

There are two versions of the theory, projectable with $N(t)$ and non-projectable with $N(x,t)$.

$\mathcal{L}_K = \sqrt{\gamma}\, N(K_{ij}K^{ij} - \lambda K^2)$ is the kinetic part of the action, $K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - 2\nabla_{(i}N_{j)})$. The potential part is more arbitrary, $\mathcal{L}_V = \sqrt{\gamma}\, N\, V(\gamma^{ij}, \nabla_i)$. Horava chooses $(\nabla_i R^{(3)})^2$. Could have $(R^{(3)})^2$. These dominate in UV. Want $\xi R^{(3)} + \Lambda$ in the IR.

These coefficients will run. Horava claims the theory is GR in the IR and is UV complete.

But I shall show there an extra degree of freedom, a strongly coupled scalar field, which makes the theory not work.

The logic applies in the general case, but for concreteness choose $\mathcal{L}_V = \sqrt{\gamma}\, N[(R^{(3)}) + \eta R^{(3)}_{ij} R^{(3)ij}]$ and the non-projectable case,
$\frac{\delta}{\delta N} \to \mathcal{H} = -K_{ij}K^{ij} + K^2 + \cdots$, $\frac{\delta}{\delta N_i} \to \mathcal{H}^i = \nabla_j K^{ij} - \lambda \nabla^i K$, $\delta\gamma^{ij} \to \partial_t K^{ij}$.
$\mathcal{H} = f(\dot{K}, \gamma, N) = f(N, K, \gamma) = 0$. This is the new equation. [Carlip: This shows it is not a first class constraint, as it is in GR.]

Set $N_i = 0$ to fix some of the gauge freedom. Then γ_{ij}, K_{ij}, N gives 13. Constraints $\rightarrow 4+1 \overset{\mathcal{H}}{\swarrow}$. Residue ~~gauge~~ gauge invariance $\rightarrow 3$. This leaves 5, or 2.5 degrees of freedom. The extra degree of freedom has an equation first order in time.

We take the equations of motion, linearize around a background, and find the ~~degree~~ equations of motion.
$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}$, $K^{ij} = \bar{K}^{ij} + k^{ij}$, $N = \bar{N} + n$, with overbars for backgroun
Constraint is $\nabla^i (N^2 \nabla_i k) = \nabla_i (2\bar{N}\, n \nabla^i \bar{K} + \bar{N} \nabla^i k) = 0$. If $\bar{K}$ is homogeneo this gives an equation for k, but it only holds at the linear level
Take $\omega, p \gg \frac{1}{L} \sim (R)^{1/2} \sim K$. $\omega \sim p \rightarrow 2$ modes that are grav. waves
$\omega \sim (pL)^2 p \gg p$, $\omega = \frac{p^4}{(1-3\lambda) p^i \partial_i \bar{K}} + \delta\omega$. $\lambda = \frac{1}{3}$ makes the theory conformally invariant, but then one does not get GR. To next order, $\delta\omega$ is imaginary. The computation of $\delta\omega$ is very complicated.

Go to the Stuckelberg procedure. Covariantize by adding compensators. For the Proca field, $F_{\mu\nu}^2 + m^2 A_\mu^2$ breaks gauge invarian Equations of motion are $\partial^\mu F_{\mu\nu} + m^2 A_\nu = 0$, $\partial^\mu A_\mu = 0$. Add ϕ and take $F^2 + m^2 (A_\mu + \partial_\mu \phi)^2$. $\delta A_\mu = \partial_\mu \varphi$, $\delta\varphi = -\varphi$. $\partial^\mu F_{\mu\nu} + m^2 (A_\mu + \partial_\mu \phi) = 0$, $\partial^\mu [A_\mu + \partial_\mu \phi] = 0$. In unitary gauge, $\phi = 0$.

For Horava gravity, set $\phi = t$: $\phi(x,t)$. Want Horava grav in unitary gauge $\phi = t$. $t \mapsto t(\tilde{t}) \rightarrow \phi \mapsto f(\phi)$. $\partial_\mu \phi \neq 0$.
Normal to hypersurface is $u_\mu = \frac{\partial_\mu \phi}{\sqrt{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}}$. $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$.
$K_{\mu\nu} = \perp_\mu{}^\rho \nabla_\rho u_\nu$. ${}^3R = \perp\perp\perp\perp R^{\prime\prime\prime}{}_1$. The action is
$S = \frac{M_P^2}{2} \int [\sqrt{-g}\, {}^{(4)}R + (1-\lambda) K^2 + \perp^{\mu\rho} \perp^{\mu\sigma} R_{\mu\nu} R_{\rho\sigma}]$, a scalar-tensor theory.
The acceleration $u^\lambda \nabla_\lambda u^\rho$ is not in the action. Now one must ensure that the equations of motion reduce to the ones before in the unitary gauge. $\frac{\delta}{\delta g^{ij}}\Big|_{\phi=t}$. $\frac{\delta}{\delta\phi} = \nabla^\mu J_\mu = 0$. $\nabla^i [(1-\lambda) K^{\cdot\cdot}\ ] = 0$.
γ_{ij}, K_{ij}, $\phi \rightarrow 13$, constraint $\rightarrow 4$, residue gauge freedom $\rightarrow 4$, leaving 5.

$$\mathcal{L}_\phi = K^2\sqrt{g} = \sqrt{g}\left(\Box\phi - \frac{\nabla^\mu\nabla^\nu\phi}{X}\nabla_\mu\nabla_\nu\phi\right)^2(\lambda-1).$$

Need to specify 4 initial conditions for ϕ, but we have 5. For $\phi = \bar{\phi} + \chi$, one gets first-order in time equation for χ. $\ddot{\varphi} + (-1)^z\Delta^z\varphi = 0$. Set $\varphi, \dot{\varphi}$ with $\varphi = 0$ as $|x| \to \infty$. For a nonrelativistic theory like this, there are preferred frames where the Cauchy data simplifies. Equations of motion are $\frac{\delta}{\delta\chi} \Rightarrow \nabla^\mu J_\mu = 0$ where $J^\mu \partial_\mu \chi$

Now I shall go to the pathologies we found, such as very fast instabilities. Take $1-\lambda \ll 1$, $(1-\lambda)M_P^2$ fixed, $M_P^2 \to \infty$, leading to decoupling. Then the equation of motion for ϕ is $\lambda\nabla^i K\partial_i\chi + \bar{N}\Delta^2\chi + 6\nabla^i N\nabla_i\Delta\chi$ with $\phi = t + \chi$. Then one gets the dispersion relation $\omega \sim \frac{p^4}{\lambda p^i\partial_i K(1-3\lambda)} + i(pL)p$.
There will be some part of the background where pL will be positive or negative, leading to an instability.
For $K \sim \frac{1}{L}$, $(1-\lambda)\left[L^{-2}v^i\dot{\chi}\partial_i\chi + (\Delta\chi)^2 + \lambda(\Delta\chi)^2\right]$
Canonically normalize by $t \mapsto \hat{t} = tL^2$, $\chi \mapsto \hat{\chi} = L^{-1}(1-\lambda)^{1/2}M_P$.
Cutoff for momentum $\Lambda \sim L^{-3/4}(1-\lambda)^{1/3}M_P^{1/4}$, which goes to 0 when one goes to the GR limit $\lambda = 1$ or when the background becomes homogeneous or $L = \infty$. [Carlip: What happens if you just set $\lambda = 1$?] This problem will be present always.

In the projectable case, $N(t)$, one does not have the Hamiltonian constraint but only an averaged Hamiltonian constraint. There is an instability that is equivalent to the development of caustics.

There is a way in which this theory may be well behaved. Make the scalar field alive and see what happens to the theory.

Larry Ford, "Quantum Instabilities of de Sitter Spacetime"

de Sitter spacetime is a solution of Einstein's equations with a positive cosmological constant. Global de Sitter is maximally symmet. A portion describes inflation.

Why is the stability of deSitter space important? Instabilities might alter the predictions of inflationary models. Instabilities might lead to a natural resolution of the cosmologica constant problem. Unfortunately, decaying cosmological constant models (e.g., Dolgov, Barr, LF etc.) have been unsuccessful so fa

Part 1: Gravitons in de Sitter spacetime. $g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$

TT gauge: $h^{\mu\nu}{}_{;\nu} = 0$, $h = h^\mu_\mu = 0$, $h_{\mu\nu} u^\mu = 0$, u = comoving observer 4-velocity. Only 8 independent restrictions, so this fixes the gauge once u is chosen.

Tensor modes behave as massless scalars (Lifshitz 1946): $\Box_S h^\mu_\nu = 0$. Consequences: deSitter space is classically stable. Gravitons are equivalent to a pair of massless scalar fields $\Box_S \varphi = \varphi^{;\nu}{}_{;\nu} =$ $\Box_S h^\mu_\nu = \frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} \sqrt{-g}\, \partial^\nu h^\alpha_\beta) = 0$.

IR divergences: $\langle \varphi(x) \varphi(y) \rangle$ is not defined in the deSitter invariant state (Bunch-Davies) vacuum, but can be defined in a class of state which break deSitter symmetry (Linde, Star, Vil, LF) Consequence: linear growth in time. Is this growth an instability of deSitter space? One loop level: No, gauge invariant quantities do not grow (LF 1985). Two loop level: Controversial.

Tsamis & Woodard (1996) claim to find cosm. const. damping. $\Lambda_{\rm eff} = \Lambda(1 - \pi^{-2} l_p^4 H^6 t^2)$, $\Lambda = 3H^2$. This result disputed by others (Garriga & Tanaka). They claim it is a gauge-dependent result. [Bill Unruh: The gauge-invariant quantities are constants of the motion.]

An alternative model: gravitons coupled to photons (J. T. Hsiang, D.S. Lee, H. L. Yu & LF). Preliminaries: stress tensor renormalization in curved spacetime. Add R^2 and $R_{\alpha\beta}R^{\alpha\beta}$ counterterms in the grav action and write Einstein's eqs. as

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \alpha_0 H^{(1)}_{\mu\nu} + \beta_0 H^{(2)}_{\mu\nu} = 8\pi G_0 \langle T_{\mu\nu} \rangle .$$

Remove the divergent parts of $\langle T_{\mu\nu} \rangle$ by a renormalization of $G_0, \Lambda_0, \alpha_0, \beta_0$. We want the renormalized values of $\alpha_0, \beta_0 = 0$ to avoid a fourth-order equation. In general, $\langle T_{\mu\nu} \rangle_{ren}$ is not expressible in terms of geometric quantities.

An exception: conformally invariant fields (e.g., photons) in a conformally flat spacetime (e.g. deS) (in the vacuum state).

$\langle T_{\mu\nu} \rangle_{ren} = C B_{\mu\nu}$, $B_{\mu\nu} = \frac{1}{2} R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} + \frac{2}{3} R R_{\mu\nu} - R^{\rho}_{\mu} R_{\rho\nu} - \frac{1}{4} R^2 g_{\mu\nu}$, $C = \frac{31}{1440\pi^2}$ for photons. $B^{\mu}_{\mu} \neq 0$ conformal anomaly. In deSitter, $\langle T_{\mu\nu} \rangle_{ren} \propto g_{\mu\nu}$ so just shifts Λ. But what happens when we perturb deSitter spacetime? In general it won't be conformally flat, so we won't know $\langle T_{\mu\nu} \rangle_{ren}$: it will have nonlocal terms. However, we shall just keep the same form, $\langle T_{\mu\nu} \rangle_{ren} = C B_{\mu\nu}$. $B^{\mu\nu}{}_{;\mu} = 0$ ~~for any~~ to linear order about deSitter ~~spacetime~~. All I say will only apply for linear perturbations of the background deSitter.

Key result: equation for the metric perturbations becomes $\Box_S h^{\mu}_{\nu} + 48\pi C l_p^2 H^4 h^{\mu}_{\nu} = 0$ with tachyonic mass sign and has an exponentially growing solution: $h^{\mu}_{\nu} \propto e^{16\pi C l_p^2 H^3 t}$. Gauge invariant quantities, such as the Ricci tensor, ~~also~~ ${}^{(1)}R_{\mu}{}^{\nu}$, also grow. Stewart and Walker showed ${}^{(1)}R_{\mu\nu}$ is not necessarily gauge invariant.

[Discussion of other calculations by Mottola, Anderson, and others that do not seem to show this instability when nonlocal terms are included.]

$\tau = \frac{1}{16\pi C l_p^2 H^3} \approx 10^4 \frac{E_p^2}{E_I^2} H^{-1}$, E_p = Planck energy, E_I = energy scale of inflation.

Part II: Quantum Stress Tensor Fluctuations in de Sitter Spac

(C.H. Wu, K.W. Ng & LT; also work in progress with S.P. Miao & R. Woodar

Basic idea: look at the effects of fluctuations of the vacuum electromagnetic field stress tensor.

Stress tensor and expansion fluctuations: u^α = 4-vel, $\theta = u^\alpha{}_{;\alpha}$

Raychaudhuri eq $\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}$, $R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}$

Ordinary matter = focussing. Treat this as a Langevin equation.

$T_{\mu\nu}$ fluctuations $\rightarrow$ θ fluctuations. Conservation law for a perfect flu

$\dot{\rho} + \theta(\rho + p) = 0$. Stress tensor fluctuations $\Rightarrow$ density fluctuations.

$C_{\mu\nu\alpha\beta}(x, x') = \langle T_{\mu\nu}(x) T_{\alpha\beta}(x')\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\alpha\beta}(x')\rangle$ (conf. anomaly cancels

Let $\theta = \theta_0 + \theta_1$, $\theta_0 = 3\dot{a}/a$. Assume $\sigma_{\mu\nu} = \omega_{\mu\nu} = 0$, so $\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta$

$\frac{d\theta_1}{dt} = -(R_{\mu\nu}u^\mu u^\nu)_{g=\text{quantum}} - \frac{2}{3}\theta_0\theta_1 \Rightarrow \theta_1(t) = -a^{-2}(t)\int_{t_0}^{t} \cdots$

$E(\Delta\eta, r)$ = flat space energy density correlation function $= \frac{(r^2 + 3\Delta\eta^2)^2}{4\pi^2(r^2 - \Delta\eta^2)^6}$

Treat as distributions – integrate by parts

Inflationary expansion followed by reheating and a radiation dominated solution: $a(\eta) = \frac{1}{1 - H\eta}$ for $\eta_0 < \eta < 0$, $a(\eta) = 1 + H\eta$ for $\eta > 0$

η_0 = conformal time when inflation starts. η_s = conf. time of last scatteri

Correlation function "$\langle(\frac{\delta\rho}{\rho})\rangle$" $= (1+w)^2\int_{\eta_0}^{\eta_s} d\eta_1 a(\eta_1)\int_{\eta_0}^{\eta_s} d\eta_2 a(\eta_2)(\langle\theta(\eta_1)\theta(\eta_2)\rangle - \langle\theta(\eta_1)\rangle\langle\theta(\eta_2)\rangle$

$= \int d^3k\, e^{i\vec{k}\cdot\Delta\vec{x}} P_k(\eta_s)$ (non-gaussian fluctuations), from quantum fluctuations of the EM field (or any other conformally invariant field) in its vacuum.

We are not including the quantum fluctuations of the inflaton we assume i

Result: $P_k = -\ell_p^4 (1+w)^2 \frac{k^4|\eta_0|^3}{480\pi^2} \ln^2[a(\eta_s)]$. $P_k \propto |\eta_0|^3$ grows as the duration of inflation increases – a different instability of de Sitter spac

Interpret as due to non-cancellation of anti-correlated θ fluctuations.

Interpret the sign as telling us whether the density fluctuations on a given scale are correlated or anti-correlated. Because the scale factor is changing, the anti-correlated fluctuations do not cancel.

[Bill Unruh: Mode by mode, each P_k is like an $\langle x^2 \rangle$ for a harmonic oscillator and show should be positive. You are getting a negative result for the square of a positive quantity.] If we consider this as a piece, it can reduce other pieces. It could be that something is wrong with this. [Rafael Sorkin: Perhaps you have to put in a cutoff for k.] [Don Marolf: What goes wrong at large k?] I do not know. In coordinate space, the Fourier transform of P_k gives the derivative of a delta function, so we initially ignored it. But then we realized we shouldn't take this seriously at too short a scale, since we did only a linear analysis.

If we take this result seriously, $\frac{\delta\rho}{\rho} \propto l_p^2 \left(\frac{S}{\lambda}\right)^{2/3}$,
$S = H/|\eta_0|$ = expansion factor during inflation, λ = length scale of pert.
$\frac{\delta\rho}{\rho} < 10^{-4} \Rightarrow S < 10^{37}\left(\frac{10^{16}GeV}{E}\right)^{5/3}$. Allows enough inflation to solve the horizon and flatness problems. Opens the possibility of observing QG effects as a non-gaussian, non-scale invariant component in the large scale structure.

Summary

1) deS is classically stable
2) In pure QG, it is stable at one loop level.
3) This may not hold in higher orders?
4) In a simple model with gravitons + photons, there is a hom. instability, No eternal inflation.
5) When stress tensor fluctuations are included, there is also an inhomogeneous instability.
6) The latter could produce observable features in large scale structure.

The effects we are getting are from trans-Planckian modes

Guillem Pérez-Nadal, "Stress Tensor Fluctuations in de Sitter Spacetime"
(in collaboration with Albert Roura and Enric Verdaguer)

$$F_{abc'd'}(x,x') = \langle 0| T_{ab}(x) T_{c'd'}(x')|0\rangle - \langle 0|T_{ab}|0\rangle\langle 0|T_{c'd'}|0\rangle.$$

$\tilde{G}^a_b + \Lambda\delta^a_b = \frac{1}{M_P^2}\tilde{T}^a_b$ in the Heisenberg picture. $|\Psi\rangle = |0\rangle|\psi\rangle$.

$$\langle\Psi|\tilde{G}^a_b(x)\tilde{G}^{c'}_{d'}(x')|\Psi\rangle - \langle\Psi|\tilde{G}^a_b(x)|\Psi\rangle\langle\Psi|\tilde{G}^{c'}_{d'}(x')|\Psi\rangle =$$
$$\frac{1}{M_P^4}\langle\Psi|\tilde{T}^a_b(x)\tilde{T}^c_d(x')|\Psi\rangle - \langle\Psi|\tilde{T}^a_b(x)|\Psi\rangle\langle\Psi|\tilde{T}^{c'}_{d'}(x')|\Psi\rangle.$$

My first motivation is because my advisor asked me to do this.

De Sitter $\eta_{ij}X^iX^j = 1$ in D+1. The metric of dS_D is the Minkowski metric, η_{ij}, restricted to vectors tangent to the hyperbolo
dS_D is the set of points at a constant distance from the origin ⇒ It is the Minkowskian analogue for a sphere. In dS_D, not all pairs of points are connected by a geodesic, but only those with $0 \le d(x,x') \le 2$ with $d(x,x')$ the Minkowskian distance. Points x' with $d(x,x') > 2$ are not connected with x by a geodesic. There are geodesics between all points timelike separated.

$z(x,x') = \eta_{ij}X^i(x)X^j(x')$, $d^2 = 2(1-z)$. No geodesic to $z < -1$.

Bitensor of type $(0,k)(0,k')$, $T_{a\ldots b'}$ $V_x \times \cdots \times V_x \times V_{x'} \times \cdots \times V_{x'} \to \mathbb{R}$

A bitensor field is a rule that assigns a bitensor to each pair of points (x,

The following bitensors are invariant under all the isometries of dS_D.

- Geodesic distance $\mu(x,x')$: length of the shortest geodesic joining x and x
- Unit vector $n_a(x,x')$, tangent to the geodesic at the point x, pointing outwar
- Parallel propagator $g_{ab'}(x,x')$: operator that parallel transports vectors from x' to x along the geodesic. Parallel transport commutes with isometrie

[Don Marolf: n_a is not well defined for antipodal points x and x'.]

Allen and Jacobson's theorem: Any de Sitter invariant bitensor is a linear combination of products of g_{ab}, $g_{a'b'}$, n_a, $n_{a'}$ and $g_{ab'}$, with coefficients that depend only on μ. In principle, this theorem applies only to points connected a geodesic.

Extension to geodesically disconnected points.

If $\sigma(x)$ is the antipode of x, $\mu(x,x') = \pi - \mu(\sigma(x), x')$, $n_a(x,x') = -n_a(x, \sigma(x'))$. These eqs are true for any spacelike geodesic. The RHS are defined for x and x' geodesically disconnected $\Rightarrow$ analytic continuation of μ and n_a to geodesically disconnected point. I can do something similar for the parallel propagator, but it is harder since the parallel propagator is not defined for antipodal points. The Allen and Jacobson's theorem also holds for geodesically disconnected points, with μ, n_a and $g_{ab'}$ defined by analytical continuation. The analytical continuation has been used for a long time, e.g. by Allen and Jacobson, using $z = \cos\mu$ analytically continued. We can do this for any maximally symmetric space but haven't done it explicitly for anti-de Sitter. n_a is the derivative of the geodesic distance and so is imaginary for timelike separation. For points x, x' not connected by a geodesic, $n_a(x,x') = -n_a(\sigma(x), x')$.

$$F_{abc'd'} = P(\mu) n_a n_b n_{c'} n_{d'} + Q(\mu)(n_a n_b g_{c'd'} + n_{c'} n_{d'} g_{ab})$$
$$+ R(\mu)(n_a n_{c'} g_{bd'} + n_b n_{d'} g_{ac'} + n_a n_{d'} g_{bc'} + n_b n_{c'} g_{ad'})$$
$$+ S(\mu)(g_{ac'} g_{bd'} + g_{bc'} g_{ad'}) + T(\mu) g_{ab} g_{c'd'}.$$

Stress tensor conservation $\Rightarrow$ 3 eqs for P, Q, R, S, T $\Rightarrow$ Only two of them must be specified in order to determine the stress tensor 2-point function. The coefficients of ill-defined terms go to zero at antipodal points.

Stress tensor two-point function for a free scalar field.

When $n \to 0$, $P, Q, R, S \sim d^{-4}$, $T \to$ const $\Rightarrow$ correlations of infinite range in the massless limit. d = Minkowski distance, which is the same as the Euclidean distance on flat slices.

In Minkowski spacetime, 2-point function $\propto \mu^{-2D}$.

The infinite range is not surprising

$G^+(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle = G(\mu) \underset{m\to 0}{\approx} \frac{c}{m^2} + O(m^0)$. P, Q, Π, S depends only on derivatives of Wightman function and so $\sim d^{-4}$, but T comes from $m^2 g_{ab}\phi^2$ term in $T_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi - m^2 g_{ab}\phi^2$

Now I shall talk about the strictly massless field.

$\phi(\eta, \Omega) = \{\text{expansion}\}$ $[\chi_{LM}, \pi_{LM}] = i\delta_{LL'}\delta_{MM'}$

$\chi_{LM} = u_{LM}a_{LM} + u^*_{LM}a^+_{LM}$ where u_{LM} are particular solutions of KG eq. $[a_{LM}, a^+_{L'M'}] = \delta_{LL'}\delta_{MM'}$. Define $a_{LM}|0\rangle = 0$. Different choices of modes give rise to different vacua. Bunch-Davies vacuum gives deS inv. Wightman function. For $m \to 0$, $u_0 \to \infty$, where $u_0 \to 0$. This is the origin of the IR divergence of the Wightman function. In the strictly massless case, $m=0$, we have to choose an alternate solution for u_0. There is no de Sitter invariant vacuum for $m=$ The rest of the modes can be chosen to be the Bunch-Davies ones with m

Something similar happens to the harmonic oscillator $\ddot{q} + \omega^2 q = 0$. As $\omega \to 0$, $q(t) = \frac{1}{\sqrt{2\omega}}(a + a^+)$ also diverges. Then free particle. More natural to write $q = q_0 + p_0 t$. Canonical commutation relation for $p, q \to p_0, q_0$. Ground state $|0\rangle$ satisfies $p_0|0\rangle = 0$. Associated wave function $\psi(q)$ is a constant $\Rightarrow$ not normalizable $\Rightarrow$ strictly speaking, $|0\rangle$ is not a state of the Hilbert space. Expectatio values of observables in this state must be understood as a limit. This limit is well defined only for observables that do not depend on q_0 that is, that depend on $q(t)$ through its derivative. The expectation value of any such observable is zero.

The Kirsten and Garriga (1993) state: One can quantize the zero mode much like a free particle. $\chi_0 = \frac{1}{\sqrt{2}}[Q + (\eta - \frac{1}{2}\sin 2\eta - \frac{\pi}{2})P]$. $a_{LM}|0\rangle = 0$ for $L > 0$, $P|0\rangle = 0$. a_{LM} associated in the Bunch-Davies vacuu

Expectation values depending on Q are not defined, and those depending on P are zero. The result you get will be the same as that of a massive field as $m \to 0$. [Don Marolf: It is a state on ~~the~~ a subalgebra]

Similarly to the case of the free particle, expectation values of observables involving Q are not well defined ~~on~~ the Kirsten and Garriga state.

$T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi$ is well defined - 2-point function P, Q, R, S, T $\sim d^{-4}$. The correlations have finite range in the strictly massless case $\Rightarrow$ Discontinuity in the stress energy tensor fluctuations at $m = 0$.

One would get the same as $m \to 0$ if one just used the expression above for T_{ab} without the mass term. T_{ab} is finite in this state. The divergences cancel.

Albert Roura, "Analytical Results for the Master Equation~~of QBM~~ Quantum Brownian Motion Models and the Solutions"

Closed Q system = system + environment.

Entanglement → non-unitary evolution, environment-induced decoherence $\hat{\rho}_r = \mathrm{Tr}_E(\hat{\rho}) \Rightarrow \langle \hat{X}(t) \rangle = \mathrm{Tr}_S(\hat{X}(t)\hat{\rho}_r)$ from reduced density matrix ρ_r of the system S in environment E.

Willis Lamb did some of this; Unruh & Zurek; Hu, Paz & Zhang

Linear Quantum Brownian Motion models.

Lagrangian $L = \frac{1}{2}M\dot{x}^2 - \frac{1}{2}M\Omega^2 x^2 + \sum_j \left(\frac{1}{2}m\dot{q}_j^2 - \frac{1}{2}m\omega_j^2 q_j^2\right) + \sum_j c_j q_j x$

Master equation $\frac{d}{dt}\hat{\rho}_r = -i[\hat{H}_R, \hat{\rho}_r] - i\Gamma[\hat{x}, \{\hat{p}, \hat{\rho}_r\}] - MD_{pp}[\hat{x}, [\hat{x}, \hat{\rho}_r]] - D_{xp}[\hat{x}, \hat{p}$

Reduced density matrix propagator $\rho_r(x_f, x_f', t_f) = \int dx_i\, dx_i'\, J(x_f, x_f', t_f; x_i, x_i', t_i)\,\rho_r(x_i, x_i', t_i)$

This is when you start with an initial product state with the environment in a gaussian state. The coefficients Γ, D_{pp}, and D_{xp} are time-dependent but depend only on the initial gaussian state of the environment. We assume an infinite number of oscillators, a continuum, but it can be generalized to a finite number. If t_i is not the initial time (when there is no entanglement), the propagator would be non-Markovian.

Wigner function $W_r(X, p, t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} d\Delta\, e^{ip\Delta/\hbar}\rho_r(X - \frac{\Delta}{2}, X + \frac{\Delta}{2}, t)$.

$$\frac{\partial}{\partial t}W_r = \{H_R, W_r\}_{KM} + 2\Gamma\frac{\partial}{\partial p}(pW_r) + MD_{pp}\frac{\partial^2}{\partial p^2}W_r - D_{xp}\frac{\partial^2}{\partial x\partial p}W_r$$

Classical-like and quantum "kinematic" properties. Example: coherent superposit
[Bill Unruh: Wojciech Zurek and I found the Fourier transform with respe to X is much more useful than the Wigner function.]

Kramers-Moore(?) bracket KM is a generalization of the Poisson brackets for nonlinear potentials for the system. However, if the system is nonlinear, there are other terms in the equation.

$

What's new?

- Explicit analytical results for the coefficients of the master equation and its solutions.
- Previous derivations missed a mathematical subtlety concerning integro-differential equations.
- Implications for systems with markedly nonlocal dissipation (e.g. subohmic environments).

Stochastic description for linear QBM models (Calzetta, A.R. & Verdaguer)

Open system dynamics in terms of a Langevin equation

$\frac{d}{dt}\begin{bmatrix}x\\p\end{bmatrix} = \begin{bmatrix}0 + \frac{1}{m}p\\ -M\Omega^2 x + -2\gamma * p\end{bmatrix} + \begin{bmatrix}0\\ \xi\end{bmatrix}$. Damping kernel: $\gamma(t-\tau) = \frac{1}{M}\int_0^\infty d\omega \frac{I(\omega)}{\omega}\cos[\omega t$

$W_r(q,t) = \langle\langle\delta($~~$q$~~ $q(t) = \Phi(t)q_0 + (\Phi * \xi)(t)$. Noise kernel:

$N(t,\tau) = \int_0^\infty d\omega\, I(\omega)\coth\left(\frac{\omega}{2T}\right)\cos[\omega(t-\tau)]\theta_0(t)\theta_0(\tau)$. Fourier transform:

$W_r(q,t) \to \mathcal{W}_r(k,t)$ (characteristic function, easily giving correlation functions

$W_r(q,t) = \langle\langle\delta(q(t)-q)\rangle_\xi\rangle_{q_i}$, $q = \begin{bmatrix}x\\p\end{bmatrix}$, $\xi = \begin{bmatrix}0\\ \xi\end{bmatrix}$, $\dot{q} = L q + \xi$.

$\langle\ldots\rangle_\xi \equiv [\det(2\pi N)]^{-\frac{1}{2}}\int D\xi \ldots e^{-\frac{1}{2}\xi\cdot N^{-1}\cdot\xi}$, $\langle\ldots\rangle_{q_i} \equiv \int dq_i \ldots W_r(q_i,t)$

$(A\cdot B)(t) \equiv \int_0^t d\tau\, A(\tau)B(\tau)$. $(A * B)(t) \equiv \int_0^t d\tau\, A(t-\tau)B(\tau)$.

$\xi(t)\xi(t') = N(t-t')$. We showed that just using the path integral for the Wigner function gives the Langevin equation

$\ddot{x}(t) + \gamma * \dot{x} + \Omega^2 x = \xi$, $\gamma * \dot{x} = \int_0^t dt'\,\gamma(t-t')\dot{x}$

Time evolution of the reduced density matrix (solution of the master equation) $\mathcal{W}_r(t,k) = \int dq\, e^{-ik^T q}\langle\langle\delta(q-q(t))\rangle_{q_0}\rangle_\xi$

$= \langle e^{-ik^T\Phi(t)q_0}\rangle_{q_0}\langle e^{-ik^T(\Phi * \xi)(t)}\rangle_\xi = \mathcal{W}_r(0,\Phi^T(t)k)\, e^{-\frac{1}{2}k\sigma_T(t)k}$.

Additional information unavailable from the master equation for "non-Markovian" evolution: $\frac{1}{2}\langle\{\hat{q}(t_1),\hat{q}(t_2)\}\rangle = \langle\langle q(t_1)q(t_2)\rangle_{q_0}\rangle_\xi$

$= \langle(\Phi(t_1)q_0)(\Phi(t_2)q_0)\rangle_{q_0} + \langle(\Phi * \xi)(t_1)(\Phi * \xi)(t_2)\rangle_\xi$, correlations involving different times. [Rafael Sorkin: Can you get all the information about the original system from the Langevin equation?] I think so.

Master equation: derivation. Derive w.r.t. time and use Langevin eq.

$\frac{\partial}{\partial t} W_r(q;t) = -\nabla_q^T \langle\langle (\int_0^t d\tau\, L(t,\tau) q(\tau) + \xi(t)) \delta(q(t)-q) \rangle_\xi \rangle_{q_0}$ final bndy condition

$q(\tau) = \Phi(\tau,t) q(t) + \int_0^t d\tau\, \Phi_q(\tau,\tau') \xi(\tau')$, $\Phi(\tau,t) = -\Phi(\tau)\Phi^{-1}(t)$.

Final result: $\frac{\partial}{\partial t} W_r(q;t) = \{\nabla_q^T H(t) q + \nabla_q^T D(t) \nabla_q\} W_r(q;t)$,

$H(t) = -\int_0^t d\tau\, L(t,\tau)\Phi(t,\tau)$

Solution with final boundary conditions:

$q(\tau) = \Phi(\tau,t) q(t) + \int_0^t d\tau\, \Phi_q(\tau,\tau')\xi(\tau')$. $\Phi_q(\tau,\tau') = -\Phi(\tau,t)\Phi(t-\tau') + \theta(\tau-\tau')\Phi(\tau-\tau')$ is not advanced. The retarded matrix propagator does not factorize. If $\Phi(t-\tau) = \Phi(t,\tau)\theta(t-\tau)$, then $\Phi(t)\theta(t-\tau)$ would be a solution, but it is not. Similar problem in previous derivations (either final or mixed boundary conditions employed). Integro-differential equations are more subtle than you might have thought.

Master equation: solutions

- Fourier transform: $(\frac{\partial}{\partial t} + k^T H \nabla_k) W_r = -k^T D k W_r$
- Method of characteristic curves: $\frac{d}{d\tau} k^T(\tau) = +k^T(\tau) H(\tau)$.
- $H(\tau) = -\dot{\Phi}(\tau)\Phi^{-1}(\tau) \rightarrow \Phi_k^T(\tau) = \Phi^{-1}(\tau)$.
- Solution: $W_r[t,k] = W_r[0, \Phi\, k]$ ...

Explicit analytical results. Generic late-time limit

- Ohmic environment with finite cut-off: $I(\omega) = \frac{2}{\pi} M \gamma_0 \omega [1 + (\frac{\omega}{\Lambda})^2]^{-1}$, $\hat{\gamma}(s) = \frac{\gamma_0}{1+s/\Lambda}$.
- Analytic superohmic environment with large cut-off: $I(\omega) = \frac{2}{\pi} M [\gamma_0 + \sum_{n=1}^{\infty} \gamma_{2n} (\frac{\omega}{\Lambda})^{n}$
- Subohmic environment $I(\omega) = \frac{2}{\pi} M \Gamma \sqrt{\omega_0 \omega}$ (no cut off). det $\Phi(t)$ changes sign!

The phase space collapses to a point and then changes sign. But the coefficients of the diffusion become infinite, so there might be diffusion to avoid violating the uncertainty principle.

Applications: Environment-induced decoherence, position and momentum uncertainties

Renaud Parentani, "Correlation Patterns Associated with Black Hole Radiation (in BEC)"

This is Part II. I shall mainly be interested in the correlations and their relation to the fluxes.

NR 2nd quantized treatment. At low temperatures a large fraction of atoms condensates. $\hat{\Psi}(t,x) = \Psi_0(t,x) + \hat{\chi}(t,x) = \Psi_0(t,x)(\hat{1} + \hat{\phi}(t,x))$. Expand in ϕ. 0th order is condensate. 1st order is linear perturbation of the condensate.

$\hat{\phi}_\omega = \hat{a}_\omega e^{-i\omega t} \hat{\phi}_\omega(x) + a_\omega^\dagger e^{+i\omega t} \hat{\phi}_\omega^\dagger(x)$. $\hat{a}_\omega$ destroys a phonon.

$(\omega - v_0 k_\omega(x))^2 = c^2 k_\omega^2 + \left(\frac{k_\omega^2}{2m}\right)^2$. Critical frequency $\Omega = \frac{\hbar}{m}$

$\hat{\phi} = \int_0^\infty d\omega \left(e^{-i\omega t} \hat{\phi}_\omega(x) + e^{i\omega t} \hat{\varphi}_\omega^\dagger(x)\right)$.

1. When $\omega > \omega_{max}$: 2-mode sector. $\hat{\phi}_\omega = \hat{a}_\omega^{u,in} \phi_\omega^{u,in}(x) + \hat{a}_\omega^{v,in} \phi_\omega^{v,in}(x)$ = same with in → out.

2. When $\omega < \omega_{max}$: 3-mode sector. $\hat{\phi}_\omega = \hat{a}_\omega^u \phi_\omega^u + \hat{a}_\omega^v \phi_\omega^v + \hat{a}_\omega^{u\dagger} (\varphi_{-\omega}^u)^*$ (both in in and out basis) ⇒ 3x3 Bogolubov transformation

$\phi_\omega^{u,in} = \alpha_\omega \phi_\omega^u + B_{-\omega} (\varphi_{-\omega}^u)^* + A_\omega \phi_\omega^v$

$\phi_\omega^{v,in} = \alpha_\omega^v \phi_\omega^v + \tilde{B}_\omega (\varphi_{-\omega}^u)^* + \tilde{A}_\omega \phi_\omega^u$

$\phi_{-\omega}^{u,in} = \alpha_{-\omega} \phi_{-\omega}^u + \beta_\omega (\varphi_\omega^u)^* + \tilde{B}_\omega (\varphi_\omega^v)^*$

$|B_\omega|^2$ can differ from $|B_{-\omega}|^2$ (because 3x3). This is because in BEC, a white hole is much more similar to a black hole. The white hole horizon in the presence of dispersion is regular and gives Hawking radiation. WH fluxes vs BH fluxes: BdG eq. is invariant under $v_0 \to -v_0$, $k \to -k$ $(\partial_x \to -\partial_x)$.

in $\begin{cases} \text{left} \\ \text{right} \end{cases}$ ⟺ out $\begin{cases} \text{right movers} \\ \text{left movers} \end{cases}$ ⇒ $\bar{n}_{-\omega}^{WH} = \bar{n}_{-\omega}^{BH}$, $\bar{n}_\omega^{WH} = |B_{-\omega}|^2 \pm \bar{n}_\omega^{BH}$, $n_\omega^{v\,WH} = |\tilde{B}_\omega|^2 \pm \bar{n}_\omega^{v\,BH}$.

Lessons: 1. Single WH, in dispersive theories,
- are stable (BdG eq.), - emit a steady radiation, - their spectra are in 1-1..

I dont which is easier to experiment, white hole radiation or black hole radiation.

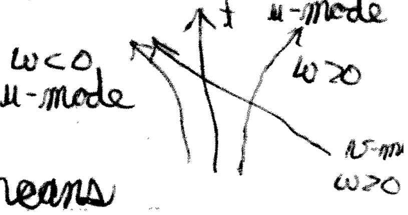
$\overline{n}_\omega \simeq \overline{n}_\omega^{(0)} = (e^{\beta\omega}-1)^{-1}$ for $\omega_{max} \gg k$, the robustness of the properties of Hawking radiation to the UV physics.

In the in-vacuum, $\overline{n}_\omega = |\beta_\omega|^2$. In a non-vacuum state,

$$\overline{n}_\omega = \overline{n}_\omega^{in} + |A_\omega|^2(n_\omega^{v\,in} - \overline{n}_\omega^{in}) + |\beta_\omega|^2(1+\overline{n}_\omega^{in}+\overline{n}_{-\omega}^{in})$$

The origin of the correlation. HR is pair production.

$|0, in\rangle = e^{\beta/\alpha\, a_\omega^\dagger a_{-\omega}^\dagger}|0, out\rangle = |0, out\rangle + \frac{\beta}{\alpha}|1_\omega, 1_{-\omega}\rangle + \dots \Rightarrow$ EPR correlations between u modes across the horizon.



What about their spacetime properties?

These correlations show up through various means

0°] wave packets of in modes

1°] Stimulated emission. Wald, Bekenstein '75, '76

2°] Correlation functions: $G_2 = \langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(x')\rangle$ Carlitz-Wiley 1987 (has a bump on the partner's trajectory) (as in inflation Campo-RP 2003)

3°] Y. Aharonov "post selection" $T_{\mu\nu}(x) = \langle \hat{T}_{\mu\nu}(x)\, \Pi_{final}\rangle$ "conditional mean value" $\in \mathbb{C}$! (Massar, RP '95).

Rel. propagation $ds^2 = -dt_{PG}^2 + (dr - v\,dx)^2$

Let's replace relativistic dispersion by BEC $\Omega^2 = c^2k^2 + \frac{k^2}{\Lambda^4}$.

Is the spacetime pattern modified? Yes and no. Same late time pattern (because phase of $\alpha_\omega^*\beta_\omega$ "unchanged").

How are both correlations and fluxes encoded in 2-point function

Flux: take $x = x' \to \infty \Rightarrow G_\omega^{in} = \frac{1}{2}(|\alpha_\omega|^2 + |\beta_\omega|^2)|\phi_\omega^{out}|^2$

Long distance correlation: take $x = -x' \to \infty$. The flux is 'diagonal' whereas L.D. correlations are 'interfering'. $\overline{n}_\omega \sim \langle 2|a_\omega^\dagger a_\omega|2\rangle \sim |\beta_\omega|^2$.

$G_\omega(x, -x) \sim \langle 0|a_\omega a_{-\omega}^\dagger|0\rangle$. In BEC: $Re(\hat{\phi}) = \hat{\chi} = \frac{\delta\rho}{2\rho_0}$: density fluctuation

3 waves $\Rightarrow$ 3 types of correlations... $\Rightarrow$ 3 coefficients $A_\omega, B_\omega, C_\omega$.

$B_\omega \doteq \mathrm{Tr}\left[\hat{\rho}_{in}\, a_\omega^{out,u}\, a_{-\omega}^{out,u}\right] \in \mathbb{C} = n_\omega^{in\,u}\, \alpha_\omega \beta_{-\omega}^* + n_\omega^{in\,v} A_\omega B_\omega^*$ $+ (n_{-\omega}^{in\,u} + 1)\, \beta_\omega^* \alpha_{-\omega}$. The first 2 are absent in the in-vacuum. The last one is amplified when $n_{-\omega}^{in\,u} \neq 0$ (<u>no</u> phase shift)

When u-v mixing small $\Leftrightarrow |A_\omega B_\omega| \ll 1 \Rightarrow \alpha_\omega \beta_{-\omega}^* = \beta_\omega^* \alpha_{-\omega} \Leftarrow$ (double amplific).

Unlike fluxes which receive additional contributions due to $n_\omega^{in} \neq 0$, correlations get amplified by a multiplicative factor $\in \mathbb{R}^+$ containing $n_\omega^{in} \neq 0 \Rightarrow$ the vacuum pattern is amplified <u>without</u> being modified. Thermal radiation hides the flux of the Hawking radiation but enhances the correlations.

The spacetime pattern. Computing $G(t,x;t,x')$ (equal t). The $\int^{\omega_{max}} d\omega$ gives constructive interference along 'correlated' characteristics. This is exactly what Carusotto (?) et al numerically obtained.

Will the signal be very weak or not? Of course, it will be very weak. Amplification by $c^2 \to 0$? Cornell '09. The correlation pattern is <u>very</u> robust, found even when $\kappa \to \infty$ or $\kappa \to 0 \Rightarrow$ still a "signature" of Hawking radiation. We put $\partial_x v = 0$ but $\partial_x^2 v = 0$ at the horizon to given an extreme horizon, $\kappa = 0$.